A Heat Transfer Textbook

Fifth Edition

Solutions Manual for Chapter 2

by

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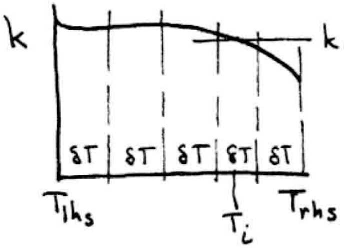
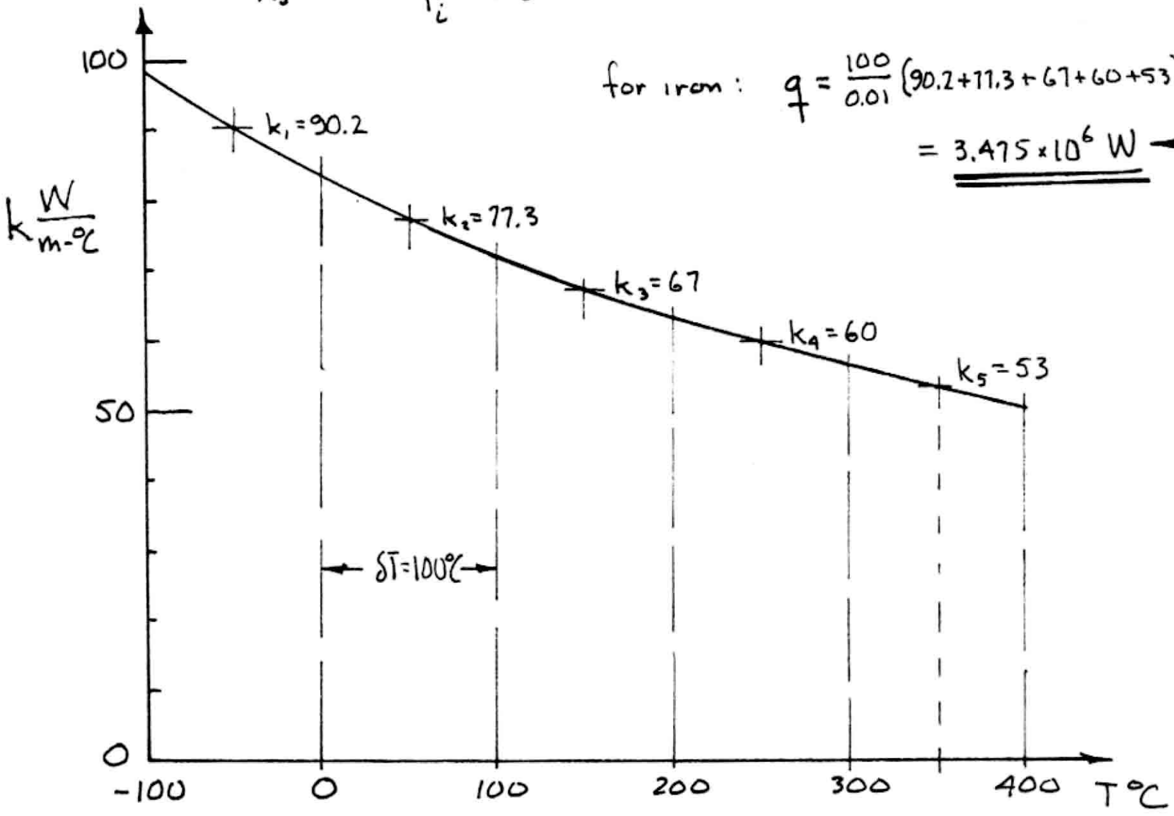
Problem 2.1 The solution of this problem is given, incidentally, in the last paragraph of the solution to Problem 1.21

Problem 2.2 Show how to evaluate q through a plane wall when k is an arbitrary function of T. Find q for a 1 cm iron wall with T_{lhs} = -100°C, T_{rhs} = 400°C.

$$q = \text{constant} = -k(T)\frac{dT}{dx} \quad \text{or} \quad \underbrace{\int_0^L q\,dx}_{qL} = -\int_{T_{lhs}}^{T_{rhs}} k(T)\,dT$$



so $$q = +\frac{\delta T}{L}\sum_i k_i(T)$$



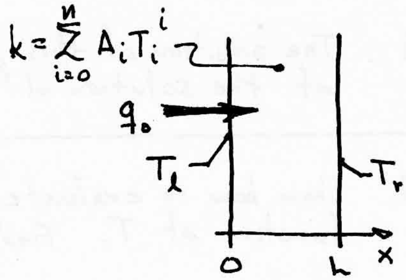
for iron: $$q = \frac{100}{0.01}(90.2+77.3+67+60+53) = \underline{\underline{3.475\times10^6\ W}}$$

If we had used k = k = (98 + 50)/2 = 74 W/m-°C we would have got:

$$Q \approx 74(500)/0.01 = \underline{3.7\text{x}10^6\ \text{W}}$$

This is 6.5% high. This relatively small error results from treating the curve above as though it were a straight line.

2.3 For the wall shown, derive $T(x)$.
Does this $T(x)$ give $q \neq q(x)$?



$$\frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \underbrace{\dot{q}}_{=0} = \underbrace{\rho c \frac{\partial T}{\partial t}}_{=0 \text{ since steady}}$$

so

$$\frac{d}{dx} k \frac{dT}{dx} = 0 . \quad \text{Integrate and get} \quad k\frac{dT}{dx} = \text{constant}$$

$$\text{but } -k\frac{dT}{dx}\Big|_{x=0} = q_o \quad \text{so} \quad \underline{k\frac{dT}{dx} = -q_o}$$

Put the $k(T)$ function in this and rearrange it:

$$-q_o\, dx = \left(\sum_{i=0}^{n} A_i T^i\right) dT \qquad \text{(eqn.1)}$$

Integrate from $T(x=0)=T_l$ to $T(x=L)=T_r$

$$\underline{\underline{-q_o x = \sum_{i=0}^{n} A_i \int_{T_l}^{T} T^i dT = \sum_{i=0}^{n} \frac{A_i}{(i+1)}\left[T^{i+1} - T_l^{i+1}\right]}}$$

This is the required function. It cannot be written explicitly in the form $T=T(x)$ if $n>1$. For $n=0$ ($k=$ constant $=A_o$) we get

$$-q_o x = A_o\,[T(x) - T_l] \quad \text{or} \quad q_o = -k\frac{T(x)-T_l}{x}$$

as we would expect. Differentiating this result gives back (eqn. 1) which can be expressed as

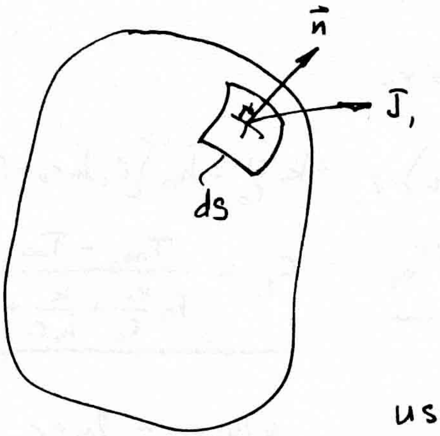
$$q_o = -\left(\sum_{i=0}^{n} A_i T^i\right)\frac{dT}{dx} = -k\frac{dT}{dx} = q$$

Thus $\underline{q = q_o \text{ which is a constant.}}$

This means that the heat flux is the same at each cross-section as we know it must be.

2.4 Combine Ficks Law with the Conservation of Mass to get a 2nd order d.e. in c_1.

$$\vec{J}_1 \frac{m^3}{m^2 s} = -\left(\mathcal{D}_{1-2} \frac{m^2}{s}\right)\left(\vec{\nabla} c_1\right) \frac{(m^3/m^3)}{m} \quad ; \quad \dot{m}_1 \frac{kg}{s} = \left(\rho_1 \frac{\partial c_1}{\partial t}\right) \frac{kg}{m^3 s} \, V m^3$$



$$-\rho_1 \int_S \vec{J}_1 \cdot \vec{n}\, dS = -\rho_1 \int_R \frac{\partial c_1}{\partial t}\, dR$$

$$\rho_1 \mathcal{D}_{1-2} \int_S \vec{\nabla} c_1 \cdot \vec{n}\, dS - \rho_1 \int_R \frac{\partial c_1}{\partial t}\, dR = 0$$

use Gauss' theorem: $\int_R \vec{\nabla} \cdot \vec{A}\, dR = \int \vec{A} \cdot \vec{n}\, dS$

Thus:

$$\rho_1 \mathcal{D}_{1-2} \int_R \left(\nabla^2 c_1 - \frac{1}{\mathcal{D}_{12}} \frac{\partial c_1}{\partial t}\right) dR = 0$$

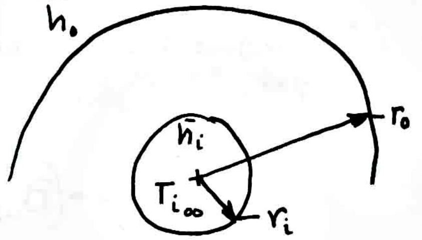
so

$$\underline{\underline{\nabla^2 c_1 = \frac{1}{\mathcal{D}_{12}} \frac{\partial c_1}{\partial t}}}$$

with this result we can solve for the concentration distribution in a region on which we know a set of b.c.s on c_1

Then we can return to Fick's law with $c_1(\vec{r}, t)$ and calculate the mass flux at any point or time.

2.5) Find the temperature distribution in the thick-walled pipe shown. T_{∞_o}



step 3) Gen'l. soln.: $T = C_1 \ln r + C_2$

step 4) $-k \left.\frac{\partial T}{\partial r}\right|_{r=r_i} = \bar{h}_i \left(T_{\infty_i} - T\right)_{r=r_i}$

$$-k \left.\frac{\partial T}{\partial r}\right|_{r=r_o} = \bar{h}_o \left(T - T_{\infty_o}\right)_{r=r_o}$$

step 5) $-k \frac{C_1}{r_i} = \bar{h}_i \left(T_{\infty_i} - C_1 \ln r_i - C_2\right)$; $-k \frac{C_1}{r_o} = \bar{h}_o \left[C_1 \ln r_o + C_2 - T_{\infty_o}\right]$

$$\underline{C_2 = \left[\frac{k}{\bar{h}_i r_i} - \ln r_i\right] C_1 + T_{\infty_i}} \qquad \underline{C_1 = \frac{T_{\infty_o} - T_{\infty_i}}{\ln \frac{r_o}{r_i} + \frac{k}{\bar{h}_o r_o} + \frac{k}{\bar{h}_i r_i}}}$$

step 6)
$$\Theta \equiv \frac{T - T_{\infty_i}}{T_{\infty_o} - T_{\infty_i}} = \frac{\ln r}{\ln \frac{r_o}{r_i} + \frac{1}{Bi_i} + \frac{1}{Bi_o}} + \frac{k/h_i r_i - \ln r_i}{\ln \frac{r_o}{r_i} + \frac{1}{Bi_i} + \frac{1}{Bi_o}}$$

or
$$\underline{\underline{\Theta = \frac{\ln r/r_i + 1/Bi_i}{\frac{1}{Bi_i} + \frac{1}{Bi_o} + \ln \frac{r_o}{r_i}}}}$$

where $Bi_i \equiv \frac{\bar{h}_i r_i}{k}$

$Bi_o \equiv \frac{\bar{h}_o r_o}{k}$

step 7) when $r = r_i$,
$$\Theta = \frac{1}{\frac{Bi_i}{Bi_i} + \frac{Bi_i}{Bi_o} + \ln \frac{r_o}{r_i}}$$

$$= \frac{1}{\frac{\bar{h}_i r_i}{\bar{h}_o r_o} + 1 + \ln \frac{r_o}{r_i}} \quad \text{(must = positive)}$$

when $r = r_o$,
$$\Theta = \frac{\ln r_o/r_i + 1/Bi_i}{\ln r_o/r_i + 1/Bi_o + 1/Bi_i} \quad \text{(also must = positive)}$$

step 8)
$$q = -k \frac{\partial T}{\partial r} = -\frac{k(T_{\infty_o} - T_{\infty_i})}{r_i} \frac{\partial \Theta}{\partial (r/r_i)} = -\underline{\underline{\frac{r_i/r}{\frac{1}{Bi_i} + \frac{1}{Bi_o} + \ln \frac{r_o}{r_i}}}}$$

2.6 Nondimensionalize the b.c.s in Problem 2.5. Let Bi's → ∞ and see that they go to b.c.'s of the first kind. Then see that that as Bi's → ∞ the

The b.c.'s become: $\frac{1}{Bi_i}\frac{\partial\Theta}{\partial(r/r_i)}\Big|_{\frac{r}{r_i}=1} = \Theta\left(\frac{r}{r_i}=1\right)$; $\frac{1}{Bi_o}\frac{\partial\Theta}{\partial r/r_i}\Big|_{\frac{r}{r_i}=\frac{r_o}{r_i}} = \Theta\left(\frac{r_o}{r_i}\right)$

As Bi_i & $Bi_o \Rightarrow \infty$ $\quad \underline{\underline{0 = \Theta\left(\frac{r}{r_i}=1\right)}}$; $\underline{\underline{0 = \Theta\left(\frac{r}{r_i}=\frac{r_o}{r_i}\right)}}$

b.c.'s of 1st kind

The solution goes to:

$$\underline{\underline{\Theta = \frac{\ln r/r_i}{\ln r_o/r_i}}}$$

same as sol'n. for b.c.'s of 1st kind in Example 2.4

2.7 Simplified explanation of the "critical radius" for a nontechnical person.

Suppose you have a hot pipe and want to keep the heat inside. You can wrap insulating material around it, the same way you wrap clothes around yourself on a cold day. If the insulating material doesn't block the heat very well, or if there's not much wind to blow the heat away from the outside surface, then something bad can happen:

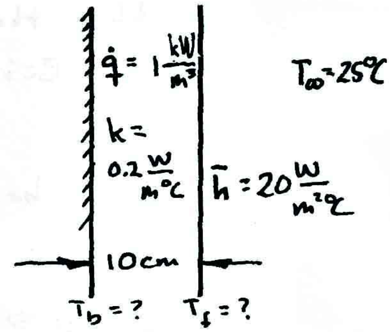
The insulation can make the outside of the pipe a lot bigger so the wind blows more heat away. This means that, in some cases, insulation can *help* heat flow. (Of course, that would only be true up to a certain thickness. Beyond that, insulation will once again keep reducing the heat flow.)

2.8 A slab which can be assumed to be two-dimensional has heat generated within it and it is cooled by convection on one side, as shown:

a) Solve for $T(x)$.

b) Evaluate T_b and T_f

c) See that $T(x)$ gives the correct q on either side.



Solution. We follow the steps in the text.

step 1) $T = T(x)$ step 2) $\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k}$ step 3) $T = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$

step 4) $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$; $\bar{h}(T - T_\infty)_{x=L} = -k\left.\frac{\partial T}{\partial x}\right|_{x=L}$ where $L = 0.1\,m$

step 5) $\underline{C_1 = 0}$; $\bar{h}\left(-\frac{\dot{q}L^2}{2k} + C_2 - T_\infty\right) = +\dot{q}L$

$$\underline{C_2 = \frac{\dot{q}L}{\bar{h}} + \frac{\dot{q}L^2}{2k} + T_\infty}$$

step 6) $T = -\frac{\dot{q}L^2}{2k}\left(\frac{x}{L}\right)^2 + \frac{\dot{q}L}{\bar{h}} + \frac{\dot{q}L^2}{2k} + T_\infty$

or $$\underline{\underline{\frac{T - T_\infty}{\dot{q}L^2/2k} = 2\frac{k}{\bar{h}L} + 1 - \left(\frac{x}{L}\right)^2}}$$ or $$\underline{\underline{\Phi = \frac{2}{Bi} + 1 - \xi^2}}$$ a

step 7) $T_b = 0 + \frac{\dot{q}L}{\bar{h}} + \frac{\dot{q}L^2}{2k} + T_\infty = 1000\left[\frac{(0.1)}{20} + \frac{(.1)^2}{0.4}\right] + 25 = \underline{\underline{55°C}}$ b

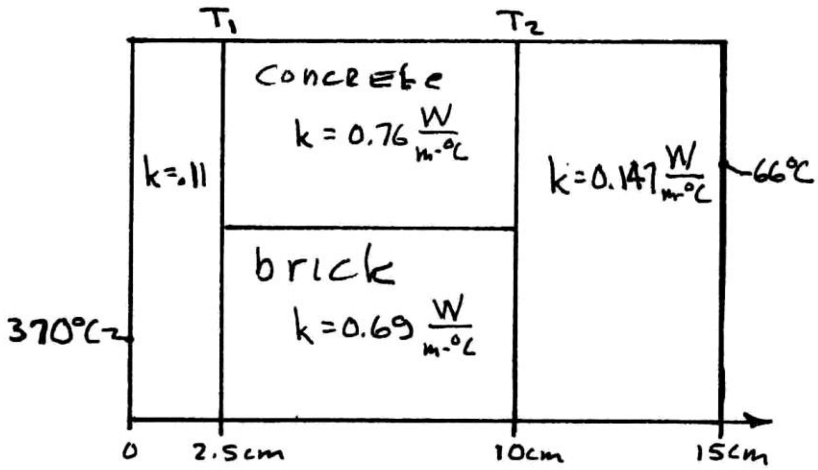
$T_f = -\frac{1000(0.1)^2}{2(.2)} + 55 = \underline{\underline{30\ °C}}$ b

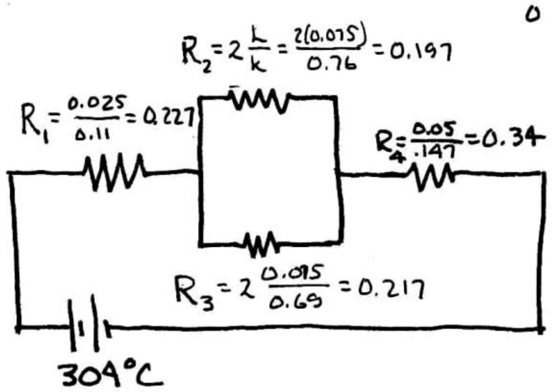
step 8) $q = -k\frac{\partial T}{\partial x}$; $q_{back} = -k\left(-\frac{\dot{q}L}{k}\left(\frac{x}{L}\right)\right)_{\frac{x}{L}=0} = \underline{\underline{0}}$ c

$q_{front} = -k\left(-\frac{\dot{q}L}{k}\left(\frac{x}{L}\right)\right)_{\frac{x}{L}=1} = \underline{\underline{\dot{q}L}}$

Thus the adiabatic b.c. is satisfied at the back face, and all the heat generated comes out the front.

2.9 Find q, T_1, T_2 for the wall shown. (Approximate heat flow one-dimensional.)





Note that since $R_1 \simeq R_2$ the one-dimensional approximation should be pretty accurate.

$$q = \frac{\Delta T}{R_1 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} + R_3} = \frac{304}{0.227 + \frac{1}{\frac{1}{0.197} + \frac{1}{0.217}} + 0.34}$$

$$\underline{\underline{q = 453.6 \ \frac{W}{m^2}}}$$

$$q = 453.6 = 0.11 \ \frac{370 - T_1}{0.025} \quad \text{so} \quad \underline{\underline{T_1 = 266.9 \ °C}}$$

$$453.6 = 0.147 \ \frac{T_2 - 66}{0.05} \quad \text{so} \quad \underline{\underline{T_2 = 220.3 \ °C}}$$

As a check, compute q through the center:

$$q = \frac{266.9 - 220.3}{\frac{1}{\frac{1}{0.197} + \frac{1}{0.217}}} = 451.2$$

which is correct within our round-off error in T_1 & T_2

Finally, the percentage of the heat flowing through the brick will be proportional to its relative resistance. So:

Percentage of heat through brick = $R_2/(R_2 + R_3) = 0.197/(0.197 + 0.217)100 = \underline{\underline{47.6\ \%}}$

2.10 Compute Q and U for the wall shown in Fig. 2.17 if $L = 0.3$ m, $A_{pine} = 0.05\ m^2/m$ into the paper, $A_{s.d.} = 0.08\ m^2/m$, $\bar{h}_\ell = 10\ W/m^2\text{-}°C$, $\bar{h}_r = 18\ W/m^2\text{-}°C$, $T_{\infty_\ell} = 30°C$ and $T_{\infty_r} = 10°C$. Let there be 5 layers each of pine and sawdust.

From the example we get the formula for U. Using the numbers above and $A = (0.05 + 0.08) = 0.13\ m^2/m$ we get:

$$U = \frac{1}{\frac{1}{18} + \frac{1}{10} \qquad \frac{1}{\frac{0.14}{0.3}\frac{0.05}{0.13} + \frac{0.06}{0.3}\frac{0.08}{0.13}}} = \underline{\underline{2.18\ \frac{W}{m^2\text{-}°K}}}$$

$$Q = UA\Delta T = 2.18\ \frac{W}{m^2\text{-}°K}\ [5(0.13\ \frac{m^2}{m})](30\text{-}10)°C = \underline{\underline{28.3\ \frac{W}{m}}}$$

2.11 Find U for the wall in Example 1.2.

$$U = \frac{1}{2R_{S.S.} + R_{cu}}$$

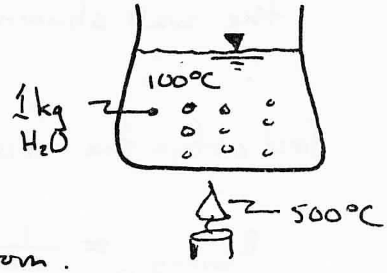
where:

$$R_{S.S.} = \frac{L}{k} = \frac{0.002\ m}{17 W/m\text{-}°C} = 0.000118\ \frac{m^2\text{-}°C}{W}$$

$$R_{cu} = \frac{L}{k} = \frac{0.003}{387} = 0.00000775\ \frac{m^2\text{-}°C}{W}$$

so $\underline{\underline{U = 4103\ \frac{W}{m^2\ °C}}}$

2.12 In the teakettle shown: $R_{t_{blng}} = 0.0002 \frac{°C\,m^2}{W}$, $R_{t_{cond}} = 0.00000625 \frac{°C\,m^2}{W}$; $R_{t_{flame}} = 0.005 \frac{°C\,m^2}{W}$; and $U = 192.1\ W/m^2\,°C$. The heated area of the bottom is $0.02\ m^2$.



a) find dT/dt when the water reaches 100°C, and find T of water in contact with bottom.

b) explain how you can boil water in a paper cup.

a) at $T_{H_2O} = 100°C$, $Q = UA\Delta T = 192.1(0.02)(400) = \underline{1537\ W}$

so:

$$Q = \frac{dU}{dt} = mc\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{1537\ J/s}{(1\ kg)(4174\ J/kg°C)} = \underline{\underline{0.36\ \frac{°C}{s}}}$$

And

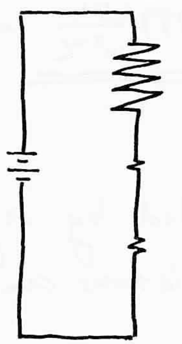
$$\underbrace{Q}_{1537} = \underbrace{\bar{h}}_{5000}\,\underbrace{A}_{0.02}\left(T_{bottom} - \underbrace{T_\infty}_{100}\right)$$

so

$$T_{bottom} = \frac{1537}{5000(.02)} + 100 = \underline{\underline{115.4\ °C}}$$

The water is superheated by 15.4 °C at the bottom.

b)



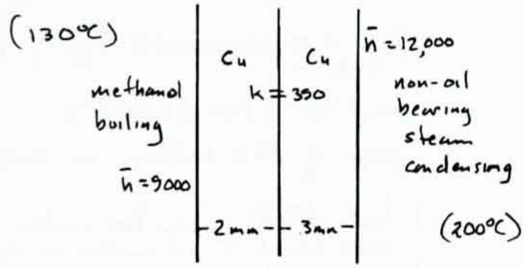
R_{flame} is large : $\Delta T_{flame\ to\ cup}$ is large

R_{paper} is small because the paper is thin: $\Delta T_{across\ paper}$ is small

$R_{boiling}$ is small: $\Delta T_{across\ boiling\ layer}$ is small

Thus the temperature difference between 100°C and the bottom of the paper is small and the paper will not be hot enough to burn.

2.13 Determine U for the wall shown



first obtain the resistances:

$$R_{\substack{\text{fouling}\\ \text{(methanol)}}} \simeq \frac{1}{5000} \ \frac{{}^\circ\text{C-m}^2}{\text{W}}$$

$$R_{\text{boiling}} = \frac{1}{9000}$$

$$R_{\substack{\text{fouling}\\ \text{(steam)}}} \simeq \frac{1}{10{,}000}$$

$$R_{\text{condensing}} = \frac{1}{12000}$$

$$R_{\text{contact}} \simeq \frac{1}{18{,}000}$$

these resistances are all the same order of magnitude -- $\simeq 10^{-4}$

$$R_{3\,\text{mm wall}} = \frac{0.003}{390} = \frac{1}{130{,}000}$$

$$R_{2\,\text{mm wall}} = \frac{0.002}{390} = \frac{1}{195{,}000}$$

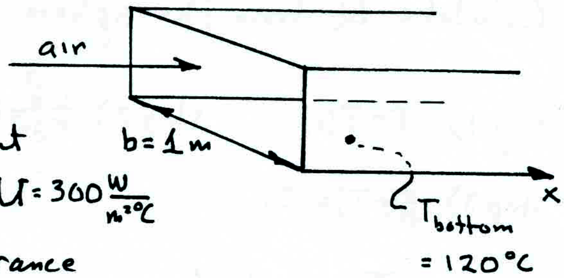
These resistances, $O(10^{-5})$ are relatively unimportant. That's to bad because they are the only ones we know with any accuracy.

Then: $$U_{\text{new}} = \frac{1}{\frac{1}{9000} + \frac{1}{12000} + \frac{1}{18000} + \frac{1}{130000} + \frac{1}{195000}} = \frac{1}{2.63\times10^{-4}} = \underline{\underline{3804 \ \frac{\text{W}}{\text{m}^2\,{}^\circ\text{C}}}}$$

$$U_{\text{used}} = \frac{1}{2.63\times10^{-4} + \frac{1}{5000} + \frac{1}{10000}} = \underline{\underline{1777 \ \frac{\text{W}}{\text{m}^2\,{}^\circ\text{C}}}}$$

Notice that very inaccurate resistances dominate both of these overall heat transfer coefficients. We could not reasonably base any precise calculations upon them.

2.14 0.5 kg/s of air at 20°C enters the channel shown. The condensation of water below the channel is done at a constant temperature of 120°C. If $U = 300 \frac{W}{m^2 °C}$



find: q and $\frac{dT}{dx}$ at the entrance

q and T, 2 m downstream

$$q_{entrance} = U \Delta T = \underline{\underline{30\ kW/m^2}}$$

$$\dot{m} c\, dT = U(b\, dx)\Delta T \quad \text{so} \quad \left.\frac{dT}{dx}\right|_{entrance} = \frac{U\, b\, \Delta T}{\dot{m} c}$$

$$= \frac{300(100)1}{\frac{1}{2}(1006)} = \underline{\underline{59.6 \frac{°C}{m}}}$$

Now from the equation above:

$$\int_{20}^{T} \frac{dT}{120 - T} = \frac{Ub}{\dot{m}c} \int_0^x dx$$

$$\ln \frac{120 - T}{120 - 20} = -\frac{Ub}{\dot{m}c} x$$

$$T = 120 - 100 e^{-\frac{Ub}{\dot{m}c}x} = 120 - 100 e^{-0.5964x}$$

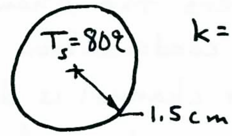
Thus, at $x = 2\ m$:

$$\underline{\underline{T_{2m} = 89.7\ °C}}$$

$$q_{2m} = U \Delta T = 300(120 - 89.7) = \underline{\underline{9090 \frac{W}{m^2}}}$$

2.15) Calulate Q from the sphere shown:

$T_\infty = 10°C$



$k = 1.28 \frac{W}{m°K}$

step 1) $T = T(r)$ Step 2) $\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) = 0$

step 3) $\frac{\partial}{\partial r}(rT) = C_1$

$$\underline{T = C_1 + \frac{C_2}{r}}$$

step 4) $T(r = r_o) = T_s$, $T(r = \infty) = T_\infty$

step 5) $C_1 + \frac{C_2}{r_o} = T_s$ $\underline{C_1} + 0 = \underline{T_\infty}$

$$\underline{C_2 = (T_s - T_\infty) r_o}$$

step 6) $\underline{T = T_\infty + (T_s - T_\infty)\frac{r_o}{r}}$

step 7) at $r = \infty$ $T \rightarrow T_\infty$ ✓

at $r = r_o$ $T \rightarrow T_\infty + T_s - T_\infty = T_s$ ✓

$$\underline{\frac{T - T_\infty}{T_s - T_\infty} = \frac{r_o}{r}} \quad \text{or} \quad \underline{\Theta = 1/\rho}$$

step 8)

$$Q = -kA\frac{\partial T}{\partial r} = -k\,4\pi r_o^2 \frac{(T_s - T_\infty)}{r_o}\frac{\partial \Theta}{\partial \rho} = 4\pi k r_o^2 (T_s - T_\infty)/r_o \rho^2$$

or $Q = 4\pi k\, r_o (T_s - T_\infty)$

and $Q = \frac{\Delta T}{\frac{1}{4\pi k r_o}}$ so $R_t = \frac{1}{4\pi k r_o} \frac{°C}{W}$

Finally $Q = 4\pi(1.28)(0.015)(70°C) = \underline{\underline{16.89\ W}}$ ← Q

2.16 Find $r_{critical}$ for an insulated sphere:

$$R_{t_{cond}} = \frac{r_o - r_i}{k 4\pi r_o r_i} \quad \text{(from Prob. 2.18) and } R_{t_{conv}} = \frac{1}{4\pi r_o^2 \bar{h}}$$

$$\frac{\partial}{\partial r_o}\left(\frac{1}{R_{t_{cond}} + R_{t_{conv}}}\right) = \frac{-\frac{1}{4\pi}\left(+\frac{1}{r_o^2 k} - \frac{2}{r_o^3 \bar{h}}\right)}{(R_{t_{cond}} + R_{t_{conv}})^2}$$

$r_{o_{critical}}$ occurs when $\dfrac{\bar{h} r_o}{k} = 2$ or when

$$\underline{\underline{r_{crit} = \frac{2\bar{h}}{k}}}$$

2.17 The heat transfer through a particular wall with $U = 225\text{W/m}^2\,°\text{C}$ is set by an overall $\Delta T = 200°\text{C}$. One layer of the wall is stainless steel with $k = 18\text{W/m-°C}$ and a thickness of 0.003 m. What is ΔT across the stainless steel layer?

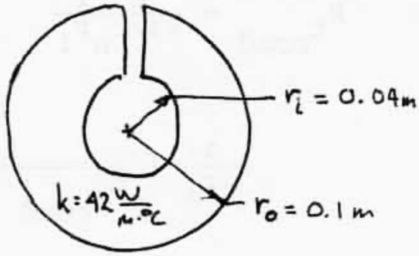
$$q = U\Delta T_{overall} = 225(200) = 45{,}000\text{W/m}^2$$

$$q = 450{,}000 = k_{s.s.} \frac{\Delta T_{s.s.}}{L_{s.s.}} = 18 \frac{\Delta T_{s.s.}}{0.003}$$

So $\Delta T_{s.s.} = \underline{\underline{75°\text{C}}}$

2.18 A hollowed-out sphere is kept at $T_{inside} = 100°C$ and $T_{outside} = 250°C$

Find Q through the shell:



step 1) $T = T(r)$ Assume radial sym.

step 2) $\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2} = 0$ & Assume steady

step 3) $T = C_1 + C_2/r$

step 4) $T(r=r_i) = T_i$; $T(r=r_o) = T_o$

step 5) $T_o = C_1 + C_2/r_o$
$\underline{T_i = C_1 + C_2/r_i}$

$$T_o - T_i \equiv \Delta T = -C_2\left[\frac{1}{r_i} - \frac{1}{r_o}\right] \quad \text{so} \quad \underline{C_2 = \frac{-r_o r_i \Delta T}{r_o - r_i}} \ ; \ \underline{C_1 = T_o + \frac{r_i \Delta T}{r_o - r_i}}$$

step 6)
$$T - T_o = -\frac{r_i \Delta T}{r_o - r_i}\left[\frac{r_o}{r} - 1\right]$$

step 7)
$$\frac{T - T_o}{T_i - T_o} = \frac{1}{\frac{r_o}{r_i} - 1}\left(\frac{r_o}{r} - 1\right) \quad \text{or} \quad \Theta = \frac{1}{\frac{1}{\rho_i} - 1}\left(\frac{1}{\rho} - 1\right)$$

Thus at $\rho = \rho_i$; $\Theta = 1$ so $T = T_i$ ✓

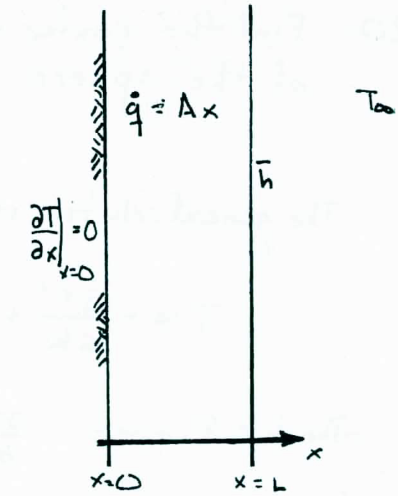
at $\rho = \rho_o = 1$; $\Theta = 0$ so $T = T_o$ ✓

step 8)
$$Q = -kA\frac{dT}{dr} = -k\,4\pi r^2\left(-\frac{r_i \Delta T}{r_o - r_i}\right)\left(-\frac{r_o}{r^2}\right) = \underline{-4\pi\frac{r_o r_i (T_o - T_i)}{r_o - r_i}k}$$

so $$Q = -4\pi\frac{0.1(0.04)(150)}{0.06}42 = \underline{\underline{-5278\text{ W}}}$$

(The minus sign means Q flows in the direction of <u>decreasing</u> r.)

2.19 Obtain $T = T(x)$ for the wall shown



$$\frac{\partial^2 T}{\partial x^2} = -\frac{\dot{q}}{k} = -\frac{Ax}{k}$$

$$\frac{\partial T}{\partial x} = -\frac{Ax^2}{2k} + C_1$$

$$T = -\frac{Ax^3}{6k} + C_1 x + C_2 \quad \text{(general solution)}$$

The b.c.'s are $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \Rightarrow \underline{C_1 = 0}$

and $\left.-k\frac{\partial T}{\partial x}\right|_{x=L} = \bar{h}\left(T - T_\infty\right)_{x=L}$

so $+k\frac{AL^2}{2k} = \bar{h}\left(-\frac{AL^3}{6k} + C_2 - T_\infty\right)$

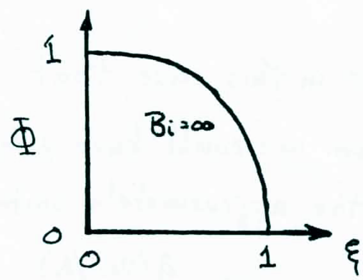
From which we get: $C_2 = +\frac{AL^2}{2\bar{h}} + \frac{AL^3}{6k} + T_\infty$

so:

$$T = T_\infty + \frac{AL^3}{6k}\left[1 - \underbrace{\left(\frac{x}{L}\right)^3}_{\xi^3} + \frac{3k}{\bar{h}L}\right]$$

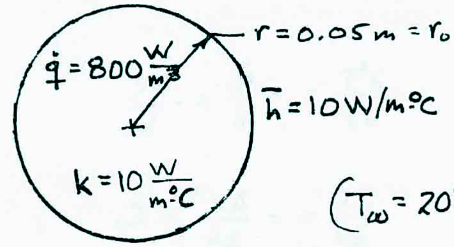
or

$$\underline{\underline{\frac{T - T_\infty}{AL^3/6k} \equiv \Phi = \left(1 + \frac{3}{Bi}\right) - \xi^3}}$$



2.20 Find the center temperature of the sphere shown:

steady state



$(T_\infty = 20^\circ C)$

The general solution is

$$T = -\frac{\dot{q} r^2}{6k} + C_1 + \frac{C_2}{r}$$

The b.c.'s are: $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \Rightarrow \underline{C_2 = 0}$

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=0.05=r_o} = \bar{h}(T - T_\infty)_{r=r_o}$$

$$+\frac{\dot{q} r_o^2}{3k} = Bi\left(-\frac{\dot{q} r_o^2}{6k} + C_1 - T_\infty\right) \quad \text{where } Bi \equiv \frac{\bar{h} r_o}{k}$$

$$\text{so:} \quad C_1 = T_\infty + \left(1 + \frac{2}{Bi}\right)\frac{\dot{q} r_o^2}{6k}$$

Thus:

$$T - T_\infty = \frac{\dot{q} r_o^2}{6k}\left[-\left(\frac{r}{r_o}\right)^2 + \left(1 + \frac{2}{Bi}\right)\right]$$

$$\text{or} \quad \boxed{\Phi = \left(1 + \frac{2}{Bi}\right) - \rho^2}$$

Finally

$$T_{\text{℄}} = T(r=0) = \left(1 + \frac{2}{Bi}\right)\frac{\dot{q} r_o^2}{6k} + T_\infty$$

$$= \left(1 + \frac{2(10)}{10(0.05)}\right)\frac{800(0.05)^2}{6(10)} + 20 = \underline{\underline{21.37^\circ C}}$$

Notice in this case that $Bi = \frac{10(0.05)}{10} = 0.05 \ll 1$ so $T_{inside} \simeq$ constant. Therefore we could have simply written $\bar{h}A(T_b - T_\infty) = \dot{q}\,\text{Volume}$ where T_b is the approximately uniform sphere temperature. Then:

$$T = \frac{\dot{q}(\text{Vol.}/A)}{\bar{h}} + T_\infty = \frac{800(r_o/3)}{10} + 20 = \underline{\underline{21.33^\circ C}}$$

Thus the full analysis was really not necessary.

Problem 2.21 Derive an expression for the thermal resistance of a spherical shell of inner radius r_i and outer radius r_o.

We give the solution to the heat conduction equation for this case in Problem 2.18. The resulting heat transfer (considering it to flow from the inside to the outside) is:

$$Q = 4\pi\, r_o r_i k\, \Delta T/(r_o - r_i)$$

The thermal resistance is then

$$R_t = \Delta T/Q = \underline{[(r_o - r_i)/4\pi r_o r_i k] \text{ K/W}}$$

2.22 Consider the hot water heater in Problem 1.11. Let it be insulated with material whose $k = 0.12\,W/m\text{-}°C$ and whose thickness $= 0.02\,m$. Then if $\bar{h} = 16\ W/m^2\text{-}°C$, Find T_m for the tank, the initial rate of cooling, the time required for the tank to cool to 40°C, and the additional heat loss resulting from 8 steel rods through the insulation if they are 0.01 m in diam.

$$T_m = \frac{\rho c V}{U A} = \frac{m c}{U A}\ ;$$ where U replaces $\bar{h}$ as the resistance

where: $$U = \frac{1}{\frac{1}{\bar{h}} + \frac{\text{thickness}}{k}} = \frac{1}{\frac{1}{16} + \frac{0.02}{0.12}} = 4.364\,\frac{W}{m^2\text{-}°C}$$

so: $$T_m = \frac{100(4190)}{4.364(1.3)} = 73{,}856\ \text{sec} = \underline{\underline{20.5\ \text{hr}}}$$

The initial rate of cooling is given by eqn. (1.19)

$$\left.\frac{dT}{dt}\right|_{t=0} = -\left.\frac{T - T_\infty}{T}\right|_{T=T_i} = -\frac{75-20}{20.5} = \underline{\underline{-2.68\,\frac{°C}{hr}}}$$

And from eqn. (1.21): $$\frac{40-75}{20-75} = e^{-t/20.5}$$

where t is the time required for the tank to reach 40°C.

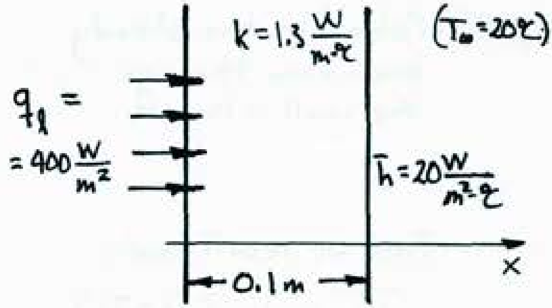
$$\underline{\underline{t = 9.266\ \text{hr}}}$$

Now, $$Q_{rods} = 8 A_{rods} \frac{\Delta T}{\frac{L}{k} + \frac{1}{\bar{h}}} = 8\pi(0.005)^2 \frac{\Delta T}{\frac{0.02}{43} + \frac{1}{6}} = 0.0100\,\Delta T$$

so $$Q_{Total} = (0.0100 + UA)\Delta T = 5.683\,\Delta T$$ and the

rods contribute only $\frac{0.00376}{5.677}\,100 = \underline{\underline{0.18\ \%}}$

2.23 Obtain a dimensionless equation for T in the slab shown. Obtain a dimensionless equation for T at the left-hand wall. Compute the left and right hand temperatures.



$$\frac{d^2T}{dx^2} = 0 \quad ; \quad T = C_1 x + C_2 \;:\; \text{general solution}$$

$$-k \left.\frac{dT}{dx}\right|_{x=0} = q_\ell \qquad : 1^{st} \text{ b.c.}$$

$$q_\ell = \bar{h}(T - T_\infty)_{x=L} \;:\; 2^{nd} \text{ b.c.}$$

so:

$$-kC_1 = q_\ell \qquad \text{so} \quad C_1 = -q_\ell/k$$

ξ:

$$q_\ell = \bar{h}\left(-q_\ell L/k + C_2 - T_\infty\right) \; ; \quad C_2 = \frac{q_\ell}{\bar{h}} + \frac{q_\ell L}{k} + T_\infty$$

Then

$$T = -\frac{q_\ell}{k} x + \frac{q_\ell}{\bar{h}} + \frac{q_\ell L}{k} + T_\infty$$

or

$$\underline{\underline{\frac{T - T_\infty}{q_\ell L/k} = \frac{k}{\bar{h}L} - \left(\frac{x}{L} - 1\right)}} \quad \text{or} \quad \underline{\underline{\Phi = \frac{1}{Bi} + (1 - \xi)}}$$

Then

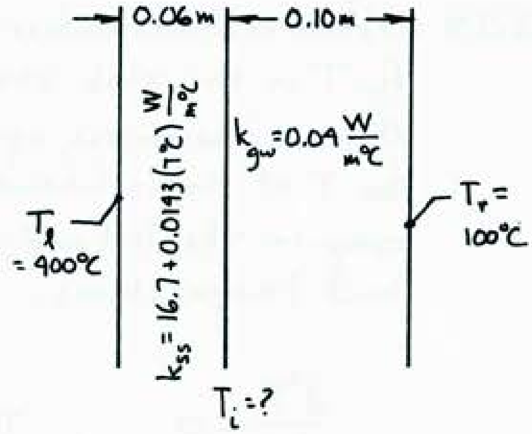
$$\underline{\underline{\frac{T_\ell - T_\infty}{q_\ell L/k} = 1 + \frac{1}{Bi}}}$$

and

$$T_\ell = T_\infty + \frac{q_\ell}{\bar{h}} = 20 + \frac{400}{20} = \underline{\underline{40°C}}$$

$$T_r = T_\infty + \frac{q_\ell}{\bar{h}} + \frac{q_\ell L}{k} = 40 + \frac{400(0.1)}{1.3} = \underline{\underline{100.8°C}}$$

2.24) Calculate the steady heat flow through the wall shown:



First we recall that:

$$\overline{k_{ss}} = k_{ss}\left(\frac{T_\ell + T_i}{2}\right)$$
$$= 16.7 + 0.00715(T_\ell + T_i)$$

Then:

$$q_{ss} = q_{gw} \quad \text{or} \quad 16.7\,\frac{T_\ell - T_i}{0.06} + 0.00715\,\frac{T_\ell^2 - T_i^2}{0.06} = 0.04\,\frac{T_i - T_r}{0.10}$$

$$\text{or} \quad 111{,}333 - 278.3T_i + 19{,}067 - 0.11917T_i^2 = 0.4T_i - 40$$

$$\text{or} \quad T_i^2 + 2339T_i - 1{,}094{,}570 = 0$$

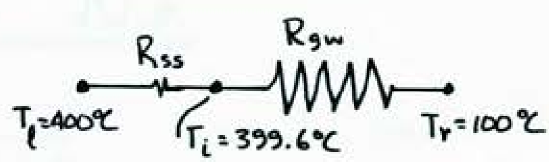
Solving this quadratic equation in T_i, we get:

$$T_i = -\frac{2339}{2} \pm \sqrt{\left(\frac{2339}{2}\right)^2 + 1{,}094{,}570} = 399.6 \text{ or } -2738^\circ C$$

The second result would violate the 3rd Law of Thermodynamics and it must be rejected. Thus:

$$\underline{\underline{T_i = 399.6^\circ C}}$$

This value of T_i is very close to T_ℓ because the steel offers far less resistance than the insulation.

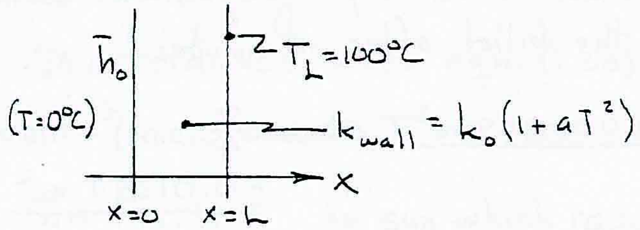


Finally:

$$\underline{\underline{q = k_{gw}\,\frac{T_i - T_r}{0.1} = 0.4(399.6 - 100) = 119.8\ \frac{W}{m^2}}}$$

(We could also have evaluated $q = q_{ss}$ but we only know ΔT_{ss} to one decimal place (0.4°C), so that calculation would be inaccurate)

2.25 Rework Problem 1.29 with a heat transfer coefficient, $\bar{h}_o = 40\,W/m^2{}^\circ C$ on the outside (i.e. on the cold side.)



general solution:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

$$k_o(1+aT^2)\frac{dT}{dx} = C_1 \equiv q$$

$$T + \frac{a}{3}T^3 = \frac{C_1}{k_o}x + C_2$$

b.c.'s: $T_L = 100°C$, $T_L + \frac{a}{3}T_L^3 = C_1\frac{L}{k_o} + C_2$

$$\bar{h}_o(T(x=0) - 0) = k(x=0)\left.\frac{dT}{dx}\right|_{x=0} = C_1 \quad \text{or} \quad T(x=0) = C_1/\bar{h}_o$$

combine the second b.c. with the general solution at $x=0$

$$\frac{C_1}{\bar{h}_o} + \frac{a}{3\bar{h}_o^3}C_1^3 = C_2$$

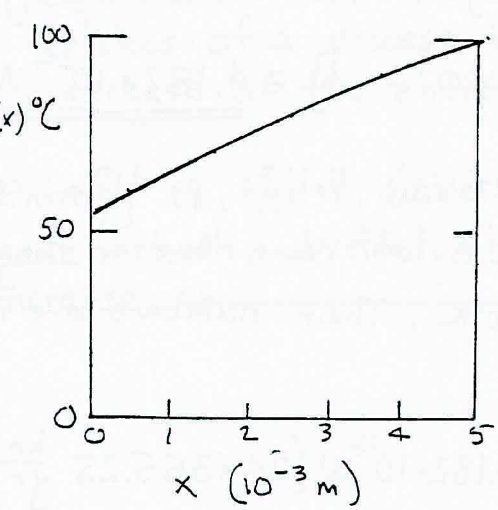
Then put this in the first b.c. to get rid of C_2

$$T_L + \frac{a}{3}T_L^3 = C_1\left[\frac{L}{k_o} + \frac{1}{\bar{h}_o}\right] + \frac{aC_1^3}{3\bar{h}_o^3} \quad \text{or} \quad 100 + 33.33 = 0.05833C_1 + 5.21\times10^{-10}C_1^3$$

so, by trial and error, $C_1 = 2200$. Then $C_2 = 133.3 - 2200\frac{0.005}{0.15} = 60$

so:

$$T + 0.0000333\,T^3 = 14{,}667\,x + 60$$



finally $q = C_1 = 2200\ \frac{W}{m^2}$

Problem 2.26 We must illuminate a Space Station experiment in a large tank of water at 20 °C. What is the maximum wattage of a submerged 3 cm diameter spherical light bulb that will illuminate the tank without boiling the surrounding water. The bulb is an LED that coverts 70% of the power to light. Bear in mind that this will occur in zero gravity.

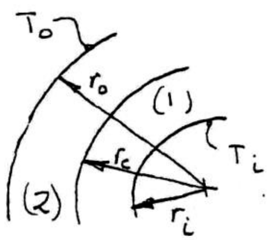
Solution The problem of heat conduction from a spherical cavity in an infinite medium is solved completely in the solution to Problem 2.5. The result is:

$$Q = 4\pi kR(T_{\text{cavity wall}} - T_\infty) = 4\pi(0.653)(0.015)(100 - 20) = \underline{\underline{9.85\ \text{W}}}$$

where we use k = 0.653 W/m-K at an average water temperature of 60°C. The power of the bulb is therefore 9.85W/0.30 = $\underline{\underline{32.8\ \text{W}}}$

Problem 2.27 A cylindrical shell is made of two layers: The inner one has an inner radius r_i and an outer radius r_c. The outer shell has inner and outer radii of r_c and r_o. There is a contact resistance, h_c, between the layers. The layers have different conductivities. $T(r = r_i) = T_i$ and $T(r = r_o) = T_o$. What is the inner temperature of the outer shell in terms of T_i and T_o?

Solution



general solutions: inner: $T_1 = C_1 \ln r + C_2$

outer: $T_2 = C_3 \ln r + C_4$

b.c.s: $T_1(r=r_i) = T_i$ so $T_1 = T_i + C_1 \ln\frac{r}{r_i}$

$T_2(r=r_o) = T_o$ so $T_2 = T_o + C_3 \ln\frac{r}{r_o}$

and: $k_1 \frac{\partial T_1}{\partial r}\Big|_{r=r_c} = k_2 \frac{\partial T_2}{\partial r}\Big|_{r=r_c}$ since heat flux must be continuous.

Therefore: $\frac{k_1 C_1}{r_c} = \frac{k_2 C_3}{r_c}$ or $C_3 = \frac{k_1}{k_2} C_1$

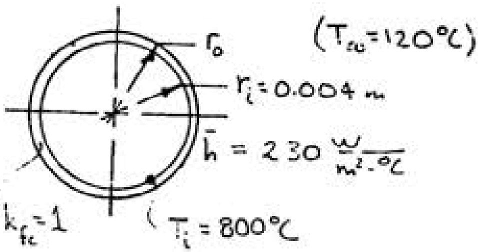
Furthermore: $h_c\left(T_1(r=r_c) - T_2(r=r_c)\right) = k_2 \frac{\partial T}{\partial r}\Big|_{r_c} = \frac{k_2}{r_c} C_1$

or (if $\Delta T \equiv T_o - T_i$)

$$\frac{h_c r_c}{k_2}\left[C_1 \ln\frac{r_c}{r_i} - \frac{k_1}{k_2} C_1 \ln\frac{r_c}{r_o} + \Delta T\right] = C_1 \; ; \quad C_1 = \frac{\Delta T}{\frac{k_2}{h_c r_c} - \ln\frac{r_c}{r_i} + \frac{k_1}{k_2}\ln\frac{r_c}{r_o}}$$

so: $\underline{\underline{T_{2_c} = T_o - C_1 \frac{k_1}{k_2} \ln r_o/r \quad \text{where } C_1 \text{ is given above}}}$

2.28 A 1 kW electric heater, 8 mm in diam. and 0.3 m long, is to be used in a corrosive environment. Therefore, it is provided with a cylindrical sheath of fireclay. The gas flows by at 120°C, and $\bar{h}$ is 230 W/m²-°C outside the sheath. The heater surface cannot exceed 800°C. Set the maximum sheath thickness and the outer temperature of the fireclay. (Hint: Use heat flux and temperature boundary conditions to get the temperature distribution. Then use the additional convective boundary condition to obtain the sheath thickness.)



general solution:

$$T = C_1 \ln r + C_2$$

b.c.'s: $T(r = r_i) = T_i = C_1 \ln r_i + C_2$ ①

& $-k_{fc} \left.\frac{dT}{dr}\right|_{r_i} = \underbrace{\frac{1000\,W}{\pi (0.008)(0.3)}}$ ②

$= \underline{q_i = 132{,}629 \frac{W}{m^2}}$

Put the general sol'n. in ② & get: $C_1 = -r_i q_i / k_{fc} = -0.004(132{,}629)/1 = \underline{-530.5°C}$

Put C_1 & gen'l. sol'n. in ① & get: $C_2 = T_i + \frac{r_i q_i}{k_{fc}} \ln r_i$

Then:

$$\underline{T - T_i = -\frac{q_i r_i}{k_{fc}} \ln \frac{r}{r_i} = -530.5 \ln \frac{r}{r_i}}$$

We must still get r_o & $T(r = r_o)$. To do so we need another b.c.

$$\bar{h}(\underbrace{T(r = r_o)}_{T_i + C_1 \ln \frac{r_o}{r_i}} - T_\infty) = -k_{fc} \left.\frac{dT}{dr}\right|_{r = r_o} = q_o = q_i \frac{r_i}{r_o}$$

So:

$$\underline{800 - 530.5 \ln \frac{r_o}{r_i} = \frac{132{,}629}{230} \frac{r_i}{r_o} = 576.6 \frac{r_i}{r_o}}$$

solve by trial and error to get $\frac{r_o}{r_i} = 1.212$ or $\underline{r_o = 0.00485\,m}$

This means the fireclay sheath cannot exceed a thickness of $r_o - r_i = 0.00485 - 0.004 = \underline{0.00085\,m \text{ or } 0.85\,mm}$ ←

Then: $T_o = T_i - 530.5 \ln \frac{r_o}{r_i} = 800 - 530.5 \ln 1.212 = \underline{\underline{698°C}}$ ←

(There is another --less desirable-- root at $r_o = 0.00822$, as well)

2.29 A very small diameter, electrically insulated heating wire runs down the center of a 7.5 mm diameter rod of 304 stainless steel. The outside is cooled by natural convection ($\bar{h} = 6.7\ \mathrm{W/m^2\text{-}^\circ C}$) in room air at 22°C. If the wire releases 12 W/m, plot T_{rod} vs. radial position in rod and give the outside temperature of the rod. (Stop and consider the physical circumstances of the problem. There are some interesting simplifications.)

first compute $T_o(r=r_o)$: $\quad q_{sfc} = \dfrac{Q}{A} = \dfrac{12}{\pi(0.0075)} = 509.3\ \dfrac{\mathrm{W}}{\mathrm{m^2}}$

$$q_{sfc} = \bar{h}(T_o - T_\infty) \quad \text{or} \quad 509.3 = 6.7(T_o - 22)$$

$$\underline{\underline{T_o = 98.0^\circ C}}$$

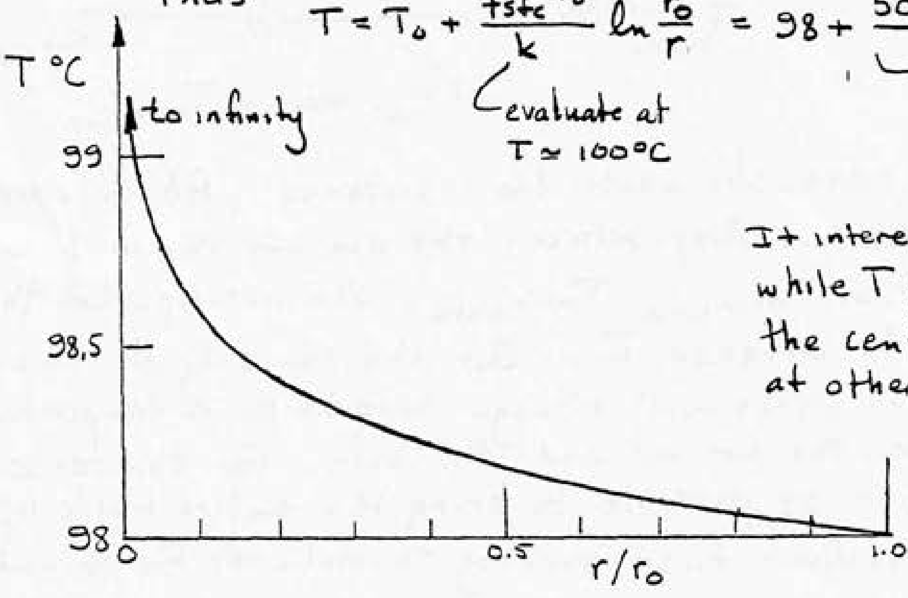
Now the general solution is $T = C_1 \ln r + C_2$. We know one b.c.: $T(r=r_o) = 98°C$. But the other b.c., $T(r=0) \Rightarrow \infty$ is singular and it won't help us. However, we also know that:

$$q_{sfc} = 509.3 = -k\left.\frac{dT}{dr}\right|_{r=r_o} = -k\frac{C_1}{r_o} \quad \text{so} \quad C_1 = -\frac{q_{sfc}\, r_o}{k}.$$

Then from the other b.c.: $\quad T_o = C_1 \ln r_o + C_2 \quad$ so $\quad C_2 = T_o + \dfrac{q_{sfc} r_o}{k} \ln \dfrac{r}{r_o}$

Thus

$$T = T_o + \underbrace{\frac{q_{sfc}\, r_o}{k}}_{\text{evaluate at } T \simeq 100^\circ C} \ln\frac{r_o}{r} = 98 + \underbrace{\frac{509.3(0.0075)}{15}}_{0.2547} \ln\frac{r_o}{r}$$



It interesting to note that while T goes to infinity at the center, the temperature at other locations remains relatively low.

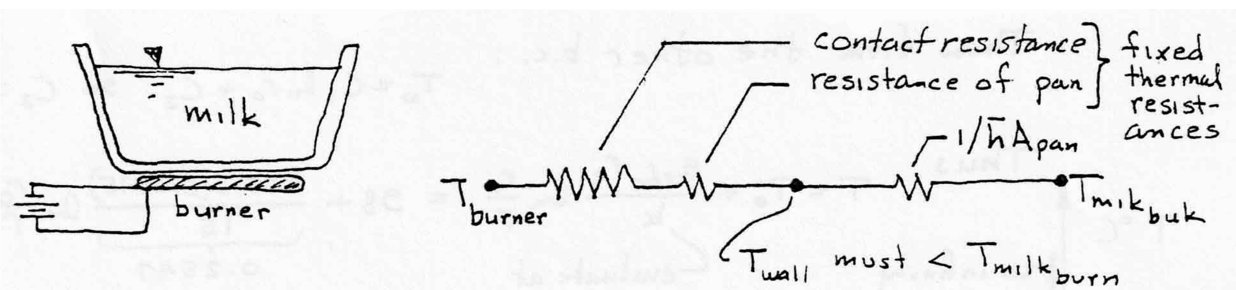
Problem 2.30 A contact resistance experiment involves pressing two slabs of different materials together, imposing a given heat flux through them, and measuring the outside temperatures of each slab. Write the general expression for h_c in terms of known quantities. Then determine h_c if: The slabs are 2 cm thick copper and 1.5 cm thick aluminum, q is 30,000 W/m^2, and the two temperatures are 15^{o}C and 22.1^{o}C.

$$q = \frac{\Delta T}{\frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2}} \quad \text{so} \quad h_c = \frac{1}{\left[\frac{\Delta T}{q} - \left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)\right]}$$

and in this case:

$$h_c = \frac{1}{\left[\frac{7.1}{30{,}000} - \left[\frac{0.02}{398} + \frac{0.015}{237}\right]\right]} = 8{,}122 \frac{\text{W}}{\text{m}^2\text{-}^\circ\text{C}}$$

Problem 2.31 A student working heat transfer problems late at night needs a cup of hot cocoa to stay awake. She puts milk in a pan on an electric stove. To heat the milk as fast as possible without burning it, she turns the stove on high and stirs the milk continuously. Use an analogous electric circuit to explain how this works. Is it possible to bring the entire bulk of the milk up to the burn temperature without burning part of it?



The student wants the resistance $1/\bar{h}A$ to decrease so the temperature drop between the pan and the milk will stay smaller than $T_{milk\text{-}burn} - T_{milk\text{-}bulk}$. She accomplishes this by stirring to increase the heat transfer coefficient. But she can do so, only up to a point. There will always have to be some temperature drop between the pan wall and the liquid bulk.

Therefore, it will never be possible to bring the milk temperature all the way up to the burning temperature, without first burning the milk at the bottom of the pan.

PROBLEM 2.32: A small, spherical hot air balloon, 10 m in diameter, weighs 130 kg with a small gondola and one passenger. How much fuel must be consumed (in kJ/h) if it is to hover at low altitude in still 27 °C air? Take $\overline{h}_{\text{outside}} = 215$ W/m^{2}K and $\overline{h}_{\text{inside}} = 126$ W/m^{2}K, as the result of natural convection. *Hint:* First determine the temperature inside the balloon that will keep it neutrally buoyant.

SOLUTION. We first calculate the temperature that must be sustained in the bag to keep it afloat.

$$\text{mass of balloon} = \text{mass of cold air} - \text{mass of hot air}$$

$$130 = \frac{\pi}{6} D^3 \left(\rho\big|_{27°\text{C}} - \rho\big|_{T \text{ inside}} \right)$$

The density inside is therefore

$$\rho\big|_{T \text{ inside}} = 1.177 - \frac{6(130)}{1000\pi} = 0.9287 \text{ kg/m}^3$$

The pressure is the same inside and outside the open-bottomed balloon, so the ideal gas law gives

$$T_{\text{inside}} = (300.15 \text{ K}) \left(\frac{1.177}{0.9287} \right) = 380.4 \text{ K} = 107.2 \text{ °C}$$

Neglecting the thermal resistance of the fabric, write $Q = UA\Delta T$ and

$$U = \frac{1}{\dfrac{1}{215} + \dfrac{1}{126}} = 79.44 \text{ W/m}^2\text{K}$$

so that

$$Q = (79.44)(100\pi)(107.2 - 27.0) = 2002 \text{ kW} = \underline{7.21 \times 10^6 \text{ kJ/h}}$$

2.33 A slab of 0.5 % carbon steel, 4 cm thick, is held at 1000°C on the back side. The front side is approximately black and radiates to black surroundings at 100°C. What is the temperature of the front side?

The general solution for this case is $T = C_1 x + C_2$ with b.c.s:

① $T(x=0) = 1273$ °K ② $-k \left.\dfrac{dT}{dx}\right|_{\text{front}} = -kC_1 = (T^4_{x=0.04} - T^4_\infty)\sigma$

$\therefore$ $\underline{1273 = 0 + C_2}$ and (using $k = 30$ for steel at 900°C)

$$-\frac{k}{\sigma} C_1 = -5.291 \times 10^8 C_1 = [0.04\, C_1 + 1273]^4 - 373^4$$

by trial and error we get: $C_1 = -3210$ so $\underline{T = 1273 - 3210x}$

$T(x=0.04) = 1273 - 128.4 = 1145$°K

$= \underline{\underline{872\text{°C}}}$ ←

(and our first guess -- that k could be evaluated at $\overline{T} = 900$°C -- should be ok.)

Problem 2.34 Use data from Fig. 2.3 to create an empirical equation for k(T) in ammonia vapor. (Be aware that, while the data form a nearly straight line, the coordinates are semi-logarithmic. The curve-fit must thus take an exponential form.) Then imagine a hot horizontal surface parallel to a cold surface a distance H below with ammonia vapor between them. Derive equations for T(x) and q, with x = 0 at the cold surface and x = H at the hot surface. Compute q if $T_{hot} = 150°C$, $T_{cold} = -5°C$, and $H = 0.15$m.

Solution We first seek an equation of the form $k = Ae^{+BT}$ to fit the almost-straight-line lnk vs. T°C data in the Fig. 2.3 coordinates. Picking two points on the graph, and solving for A and B, we get k = 0.0213exp(+0.00392T). Then:

$$\frac{d}{dx} k(T)\frac{dT}{dx} = 0 \quad \text{so} \quad k\frac{dT}{dx} = C \quad \text{so} \quad Ae^{BT}\frac{dT}{dx} = C$$

Thus: $\int Ae^{BT}dT = \int C\,dx$ or $\frac{A}{B}e^{BT} = Cx + D$

b.c. #1 : $T = T_c$ at $x = 0$ so $D = \frac{A}{B}e^{BT_c}$

b.c. #2 : $T = T_H$ at $x = H$ so $C = \frac{1}{H}\frac{A}{B}\left(e^{BT_H} - e^{BT_c}\right)$

Thus: $$\underline{T = \frac{1}{B}\ln\left[\frac{BC}{A}x + \frac{BD}{A}\right] = \underline{\frac{1}{B}\ln\left[\frac{1}{H}\left(e^{BT_H} - e^{BT_c}\right) + e^{BT_c}\right]}}$$

and $q = -k\frac{dT}{dx} = C$ so $\underline{\underline{q = -\frac{1}{H}\frac{A}{B}\left(e^{BT_H} - e^{BT_c}\right)}}$

Then:

$$q = -\frac{1}{0.15}\frac{0.0213}{0.00392}\left(e^{0.00392(350)} - e^{0.00392(-50)}\right)$$

$$\underline{\underline{q = -113.1 \text{ W/m}^2}}$$

Where the minus sign ⇒ q is opposite in sign to x (i.e. It flows downward.)

Let's make a quick rough check of this result:

$$q \simeq -k_{avg}\frac{\Delta T}{L} = -(\text{about } 0.035)\frac{400}{0.15} = 93 \text{ W/m}^2$$

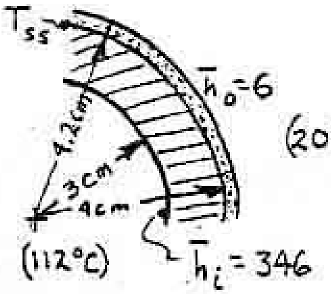
looks pretty good.

2.35 A 316 stainless steel pipe (6 cm I.D. and 8 cm O.D.). There is a 2 mm layer of 85% magnesia insulation around it. Liquid at 112°C flows inside so $h_i = 346$ W/m²-°C. The air around the pipe is at 20°C and $h_o = 6$ W/m²-°C. Calculate U based on the inside area. Sketch the equivalent electrical circuit showing all known temperatures. Discuss the results.

$$U = \frac{1}{A_i \sum R_t\text{'s}} \qquad \text{where } A_i = 2\pi r_i \frac{m^2}{m} = 2\pi(0.03) = 0.1885 \frac{m^2}{m}$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{346(0.1885)} = 0.01533\,; \quad R_o = \frac{1}{h_o 2\pi r_o} = \frac{1}{6(2\pi)(0.042)} = 0.6316$$

$$R_{s.s.} = \frac{\ln r_{ss_{out}}/r_i}{2\pi k} = \frac{\ln 4/3}{2\pi\, 15} = 0.003052 \quad R_{mag.} = \frac{\ln r_{mag}/r_{ss}}{2\pi k} = \frac{\ln 4.2/4}{2\pi(0.07)} = 0.1109$$



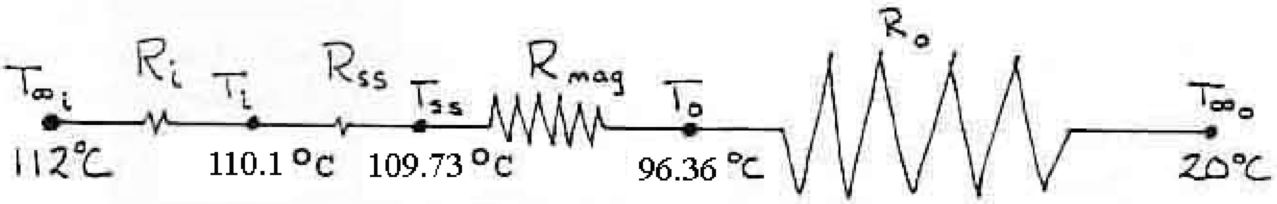
All R's have dimensions of °C-m/W

Then $\underline{\underline{U = 6.972 \text{ W/m}^2\text{-°C}}}$

& $Q = U A_i \Delta T = 6.972\,(\pi(0.06))(112-20) = \underline{120.9 \frac{W}{m}}$

Then: $T_o = T_{\infty_o} + QR_o = 20 + 120.9(0.6316) = \underline{\underline{96.36°C}}$, $T_{ss} = 96.36 + 120(0.1109) = \underline{\underline{109.73°C}}$

$T_i = 109.73 + 120.9\,(0.00305) = \underline{\underline{110.10\ °C}}$

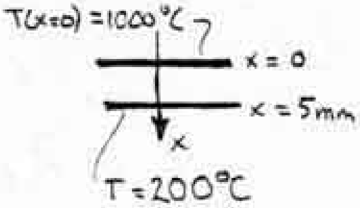


The most effective insulation here is the low outer heat transfer coefficient. The insulation would have to be much thicker to be effective. The inner heat transfer coefficient and the stainless steel offer negligible thermal resistance.

2.36 Two highly-reflecting, horizontal plates are spaced 0.005 m apart. The upper one is kept at 1000 °C and the lower one at 200 °C. There is air in between. Neglect radiation, and compute the heat flux and the mid-point temperature in the air. Use a power-law fit of the form $k = a(T\,°C)^b$ to represent the air data in Table **A.6.**

One can use various means to make the data fit. The curve fit routines in pocket calculators are recommended. Our result is

$$k = 0.0002273\,(T\,°C)^{0.8207}$$



Then: $$q = -k\frac{dT}{dx} = -aT^b\frac{dT}{dx} = \text{Constant}, C$$

integrating: $$\frac{a}{b+1}\left(T^{b+1} - 1000^{b+1}\right) = -Cx$$

(where we have included the upper b.c.)

Then: $T(x=0.005) = 1000$ so: $$-\frac{0.0002273}{0.8207+1}\left(200^{1.8207} - 1000^{1.8207}\right) = 0.005C$$

so $\underline{C = 8227}$

Thus: $$T^{1.8207} = -\frac{1.8207(8227)}{0.0002273}x + 1000^{1.8207}$$

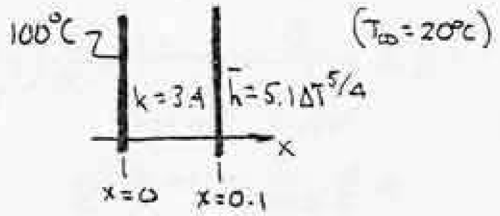
$$T = (-5.49\cdot 10^7 x + 2.90\cdot 10^5)^{0.549}$$

In the middle -- at $x = 0.0025$ m -- $\underline{\underline{T = 701.5\,°C}}$

(instead of the average value of $T = (200+800)/2 = 500\,°C$

and: $$\underline{\underline{q = C = 8227\ \text{W/m}^2}}$$

2.37 A 0.1 m thick slab with $k = 3.4$ W/m-°C is held at 100 °C on the left side. The right side is cooled with air at 20°C through a heat transfer coefficient: $\bar{h} = (5.1\ \text{W/m}\text{-}°\text{C}^{5/4})(T_{wall} - T_\infty)^{1/4}$. Find q and T_{wall} on the right.



Gen'l sol'n. $T = C_1 + C_2 x$

left b.c. $\underline{100 = C_1} + 0$

right b.c. $-k\dfrac{dT}{dx}\bigg|_{x=0.1} = 5.1(T - T_\infty)^{1/4}\Big|_{x=0.1}(T - T_\infty)\Big|_{x=0.1}$

so: $-kC_2 = 5.1(\underbrace{C_1}_{=100°C} + C_2(0.1) - T_\infty)^{5/4}$; $-C_2 = 1.5(80 + 0.1C_2)^{5/4}$

Trial & error gives $\underline{C_2 = -233\ \frac{°C}{m}}$

$\therefore$ $\underline{T - 80 = -233x}$, $T(x=0.1) = \underline{T_{wall} = 56.7\ °C}$ ←

$\underline{\underline{q = -k\dfrac{dT}{dx} = 3.4(233) = 792.2\ \text{W/m}^2}}$ ←

2.38 *Heat is generated at 54,000 W/m³ in a 0.16m diameter sphere. The sphere is cooled by natural convection with fluid at 0°C. $\bar{h} = [2 + 6(T_{wall} - T_\infty)^{1/4}]$ W/m²-°C. $k_{sphere} = 9$ W/m-°C. Find the wall temperature and center temperature of the sphere.*

$\dfrac{1}{r}\dfrac{d^2(rT)}{dr^2} = -\dfrac{\dot{q}}{k}$; $\dfrac{drT}{dr} = -\dfrac{\dot{q}r^2}{2k} + C_1$; $\underline{T = -\dfrac{\dot{q}r^2}{6k} + C_1 + \dfrac{C_2}{r}}$

b.c.'s: $\dfrac{dT}{dr}\bigg|_{r=0} = 0 = -\dfrac{C_2}{0^2}$, so $\underline{C_2 = 0}$; $-k\dfrac{dT}{dr}\bigg|_{r=0.08} = (2 + 6\Delta T^{1/4})\Delta T\Big|_{r=0.08}$

so: $+\dfrac{\dot{q}r}{3}\bigg|_{r=0.08} = \left(2 + 6\left[\dfrac{\dot{q}r^2}{6k} + C_1 - 0\right]^{1/4}\right)\left(-\dfrac{\dot{q}r^2}{6k} + C_1\right)_{r=0.08}$

or: $1440 = (2 + 6[C_1 - 6.4]^{1/4})(C_1 - 6.4)$; by trial & error, get $\underline{C_1 = 80\ °C}$

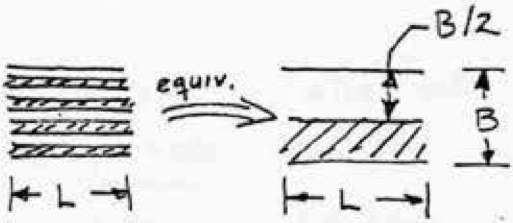
Then: $\underline{T = -1000r^2 + 80}$

$T_{℄} = T(r=0) = \underline{80°C}$ ←

$T_{wall} = T(r=0.08) = \underline{73.6°C}$ ←

2.39 Layers of equal thickness of spruce and pitch pine are laminated to make an insulating material. How should the laminations be oriented in a temperature gradient to achieve the best effect?

parallel:

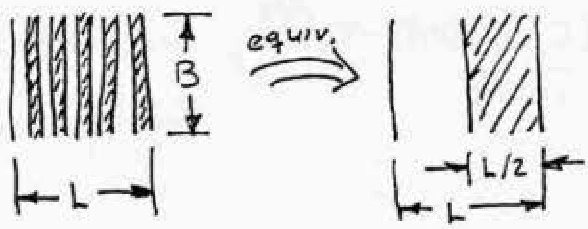


consider 1 m into paper, $A = 1 \cdot B$

$$R = \frac{1}{\frac{1}{L/k_pA} + \frac{1}{L/k_sA}} = \frac{2L}{B}\left(\frac{1}{k_p+k_s}\right)$$

$$= \frac{L}{B}\,\frac{2}{0.14+0.11} = \underline{8\,\frac{L}{B}}$$

perpendicular:



$$R = R_p + R_s$$

$$= \frac{L/2}{B\,k_p} + \frac{L/2}{B\,k_s}$$

$$= \frac{L}{B}\left(\frac{1}{2}\left[\frac{1}{0.14} + \frac{1}{0.11}\right]\right) = \underline{8.12\,\frac{L}{B}}$$

There is a very slight advantage to be gained by orienting heat flow normal to the laminations. There is about 1.5% higher resistance. ←

2.40 The resistance of a thick cylindrical layer of insulation must be increased. Will Q be lowered more by a small increase of the OD or by the same decrease in the ID.

eqn. (2.21): $Q = \dfrac{2\pi k \ell \Delta T}{\ln r_o/r_i} = \text{const.}/\ln\dfrac{r_o}{r_i}$

$$\frac{dQ}{dr_o} = c\,\frac{d(1/\ln r_o/r_i)}{dr_o} = -c\left(\frac{1}{\ln r_o/r_i}\right)^2 \frac{1}{r_o}$$

$$\frac{dQ}{-dr_o} = -c\,\frac{d(1/\ln r_o/r_i)^2}{dr_i} = -c\left(\frac{1}{\ln r_o/r_i}\right)^2 \frac{1}{r_i}$$

The lower value is obviously the greater negative number.

Therefore Q is reduced most rapidly by shrinking the inner diameter. ←

PROBLEM 2.41: You are in charge of energy conservation at your plant. A 300 m run of 6 in. O.D. iron pipe carries steam at 125 psig. The pipe hangs in a room at 25°C, with a natural convection heat transfer coefficient $\overline{h} = 6$ W/m^{2}K. The pipe has an emittance of $\varepsilon = 0.65$. The thermal resistances are such that the surface of the pipe will stay close to the saturation temperature of the steam. (a) Find the effective heat transfer coefficient between the pipe surface and the room, and the rate of heat loss from this pipe, in kWh/y. (b) It is proposed to add a 2 in. layer of glass fiber insulation with $k = 0.05$ W/m·K. The outside surface of the insulation has of $\varepsilon = 0.7$. What is the rate of heat loss with insulation? (c) If the installed insulation cost is $50/m including labor and the cost of thermal energy is $0.03/kWh, what is the payback time for adding insulation?

SOLUTION.

a) The pipe loses heat by natural convection and thermal radiation. The saturation temperature of steam at 125 psig = 140 psia = 0.963 MPa may be found from a steam table: 178.3 °C. The radiation heat transfer coefficient, with $T_m = (178.3 + 25.0)/2 = 101.6$ °C, is

$$h_{\text{rad}} = 4\varepsilon\sigma T_m^3 = 4(0.65)(5.67 \times 10^{-8})(101.6 + 273.15)^3 = 7.76 \text{ W/m}^2\text{K}$$

The effective heat transfer coefficient is $h_{\text{eff}} = \overline{h}_{\text{conv}} + h_{\text{rad}} = 6 + 7.76 = \underline{13.8 \text{ W/m}^2\text{K}}$.
The annual heat loss is

$$\begin{aligned} Q_{\text{ann}} &= h_{\text{eff}}\, \Delta T\, A(365.25 \times 24 \text{ h/y}) \\ &= (13.8)(178.3 - 25)\pi(6)(0.0254)(300)(365.25)(24) = 2.66 \times 10^9 \text{ Wh/y} \\ &= \underline{2.66 \times 10^6 \text{ kWh/y}} \end{aligned}$$

b) The surface temperature of the insulation is not known yet, but it will be much lower than the bare pipe. If we guess 40 °C, then

$$h_{\text{rad}} = 4(0.7)(5.67 \times 10^{-8})(40 + 273.15)^3 = 4.87 \text{ W/m}^2\text{K}$$

and $h_{\text{eff}} = 6 + 4.87 = 10.87$ W/m^{2}K. The heat loss is for two thermal resistances in series, with r_o is the radius of the insulated pipe:

$$\begin{aligned} R_{t_{\text{total}}} &= \frac{1}{h_{\text{eff}}(2\pi r_o l)} + \frac{1}{2\pi l k} \ln \frac{r_o}{r_{\text{pipe}}} \\ &= \frac{1}{(10.87)2\pi(3+2)(0.0254)(300)} + \frac{1}{2\pi(300)(0.05)} \ln\left(\frac{3+2}{3}\right) \\ &= 3.84 \times 10^{-4} + 5.42 \times 10^{-3} = 5.80 \times 10^{-3} \text{ K/W} \end{aligned}$$

Then

$$Q_{\text{ann}} = \frac{\Delta T}{R_{t_{\text{total}}}}(365.25)(24) = \frac{178.3 - 25}{5.80 \times 10^{-3}}(365.25)(24) = \underline{232 \text{ kWh/y}}$$

We must check whether our estimate of the surface temperature for h_{rad} is acceptable. Because the heat flow through the outside resistance equals that through the total resistance, the fraction of the temperature drop outside the insulation is

$$\frac{T_{\text{surface}} - 25}{178.3 - 25} = \frac{R_{t_{\text{outside}}}}{R_{t_{\text{total}}}} = \frac{3.84 \times 10^{-4}}{5.80 \times 10^{-3}} = 0.0662$$

Solving, the surface temperature is $T_{\text{surface}} = 35.1$ °C. Recalculating with this value gives $h_{\text{eff}} = 10.65$ W/m^{2}K, which represents a 2% reduction of the outside resistance which itself amounts to only 6.6% of $R_{t_{\text{total}}}$. There is no need to repeat the calculation with this slightly lower resistance.

We note that the increase in outside diameter after adding insulation would lower the natural convection resistance very slightly. In Chapter 8, we'll see that $\overline{h}_{\text{conv}} \sim D^{-1/4}$. For the present dimensions, $\overline{h}_{\text{conv}}$ would decrease by about 12% if insulation were added, making $h_{\text{eff}} = 9.93$ W/m^{2}K, a 9% reduction of the outside resistance, but still a very small net decrease (0.66%) in $R_{t_{\text{total}}}$.

c) The energy savings is nearly 100%: 2.66×10^6 kWh/y.

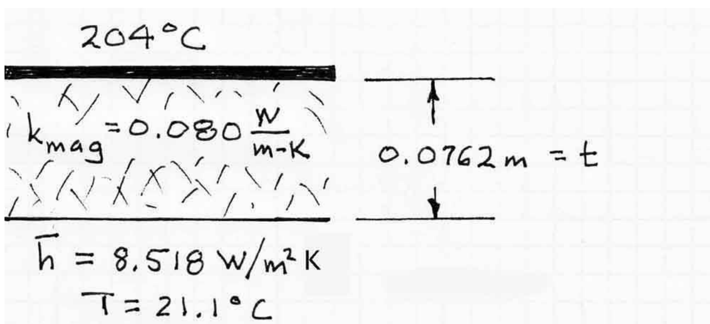
$$\text{Value of energy saved} = (2.66 \times 10^6 \text{ kWh/y})(0.03 \text{ \$/kWh}) = \$79{,}800\text{/y}$$
$$\text{Cost of insulation} = (300 \text{ m})(50 \text{ \$/m}) = \$15{,}000\text{/y}$$
$$\text{Payback time} = (15,000)/(79,800) \text{ yr} = \underline{2.26 \text{ months}}$$

Adding insulation is an excellent investment. NB: We have not included the cost of capital because the payback time is very short (the interest on $15,000 over two months will not increase the cost significantly).

Problem 2.42 A large tank made of thin steel plate contains pork fat at 400°F, which is being rendered into oil. We consider applying a 3-inch layer of 85% magnesia insulation to the surface of the tank. The average heat transfer coefficient is 1.5 Btu/hr-ft^2-°F for natural convection on the outside. It is far larger on the inside. The outside temperature is 70°F. By what percentage would adding the insulation reduce the heat loss?

Solution: We sketch a section of the tank below, with the dimensions converted to SI units for convenience (See Appendix B for conversion factors). Thus T_{inside} = 204.4°C, $T_{outside}$ = 21.1°C, the outer heat transfer coefficient is 8.518 W/m^{2}K, and the wall thickness is 0.0762 m. We get the thermal conductivity of 85% magnesia as 0.80 W/m-K directly from Table A.2.

We assume that we can neglect the resistance of the thin steel tank. We are also confident that the thermal resistance offered by the inner heat transfer coefficient is negligible. This leaves us with only two significant thermal resistances. They are the insulation, if it is present, and the outer heat transfer coefficient:



$$U = Q/A\Delta T = q/\Delta T = \frac{1}{(1/\bar{h} + t/k)}$$

$$q_{\text{with insulation}} = \frac{(201.4 - 21.2)}{\left(\frac{1}{8.518} + \frac{0.0762}{0.080}\right)} = \underline{171.3 \ W/m^2}$$

$$q_{\text{w/o insulation}} = \underline{1561 \ W/m^2}$$

Therefore, adding insulation would reduce the heat flux by (1561 -171.3)(100)/1558

or <u>89 percent</u>

Problem 2.43: The thermal resistance of a cylinder is $R_{t_{\text{cyl}}} = (1/2\pi k l)\ln(r_o/r_i)$. If $r_o = r_i + \delta$, show that the thermal resistance of a thin-walled cylinder ($\delta \ll r_i$) can be approximated by that for a slab of thickness δ. Thus, $R_{t_{\text{thin}}} = \delta/(kA_i)$, where $A_i = 2\pi r_i l$ is the inside surface area. How much error is introduced by this approximation if $\delta/r_i = 0.2$? Plot $R_{t_{\text{thin}}}/R_{t_{\text{cyl}}}$ as a function of δ/r_i. *Hint:* Use a Taylor series.

Solution.

$$R_{t_{\text{cyl}}} = \frac{1}{2\pi k l}\ln\left(\frac{r_o}{r_i}\right) = \frac{1}{2\pi k l}\ln\left(1+\frac{\delta}{r_i}\right)$$

The Taylor expansion of $\ln(1+x)$ around $x = 0$ is

$$\ln(1+x) = \sum_{n=0}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots$$

so that

$$R_{t_{\text{cyl}}} = \frac{1}{2\pi k l}\left[\frac{\delta}{r_i} - \frac{1}{2}\left(\frac{\delta}{r_i}\right)^2 + \frac{1}{3}\left(\frac{\delta}{r_i}\right)^3 - \cdots\right] \approx \frac{1}{2\pi k l}\frac{\delta}{r_i} \quad \text{for } \delta \ll r_i$$

Letting $A_i = 2\pi r_i l$, we find that, for $\delta \ll r_i$,

$$R_{t_{\text{cyl}}} \approx R_{t_{\text{thin}}} \equiv \frac{1}{2\pi k l}\frac{\delta}{r_i} = \frac{\delta}{kA_i}$$

For $\delta/r_i = 0.2$,

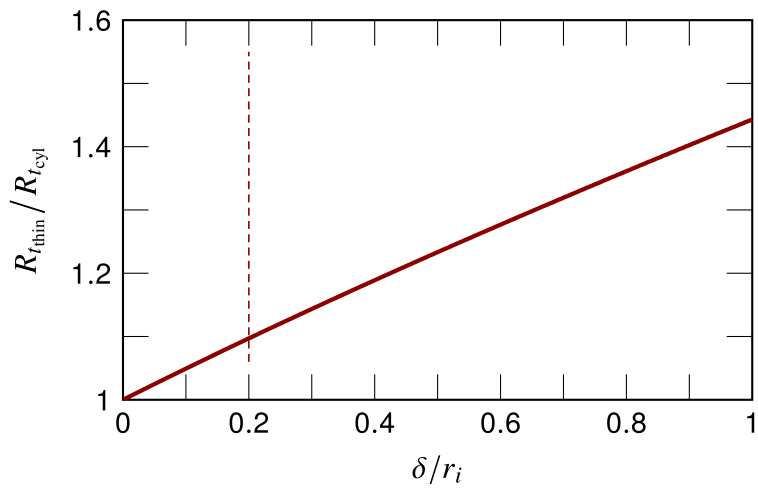
$$2\pi k l\, R_{t_{\text{cyl}}} = \ln\left(1+\frac{\delta}{r_i}\right) = 0.1832\cdots$$

and

$$2\pi k l\, R_{t_{\text{thin}}} = \frac{\delta}{r_i} = 0.2000$$

The thin wall approximation is high by 9.7% when $\delta/r_i = 0.2$.

The plot is below. To avoid numerical problems as $\delta/r_i \to 0$: a) use a few terms of the Taylor expansion of $\ln(1+\delta/r_i)$ in the denominator for $\delta/r_i < 0.15$; and b) plot only for $\delta/r_i \geq 0.001$.



Problem 2.44: A Gardon gage measures radiation heat flux by detecting a temperature difference. The gage consists of a circular constantan membrane of radius R, thickness t, and thermal conductivity k_{ct} which is joined to a heavy copper heat sink at its edges. When a radiant heat flux q_{rad} is absorbed by the membrane, heat flows from the interior of the membrane to the copper heat sink at the edge, creating a radial temperature gradient. Copper leads are welded to the center of the membrane and to the copper heat sink, making two copper-constantan thermocouple junctions. These junctions measure the temperature difference ΔT between the center of the membrane, $T(r = 0)$, and the edge of the membrane, $T(r = R)$.

The following approximations can be made:

- The membrane surface has been blackened so that it absorbs all radiation that falls on it.
- The radiant heat flux is much larger than the heat lost from the membrane by convection or re-radiation. Thus, all absorbed radiation is conducted to the heat sink, and other loses can be neglected.
- The gage operates in steady state.
- The membrane is thin enough ($t \ll R$) that the temperature in it varies only with r, i.e., $T = T(r)$ only.

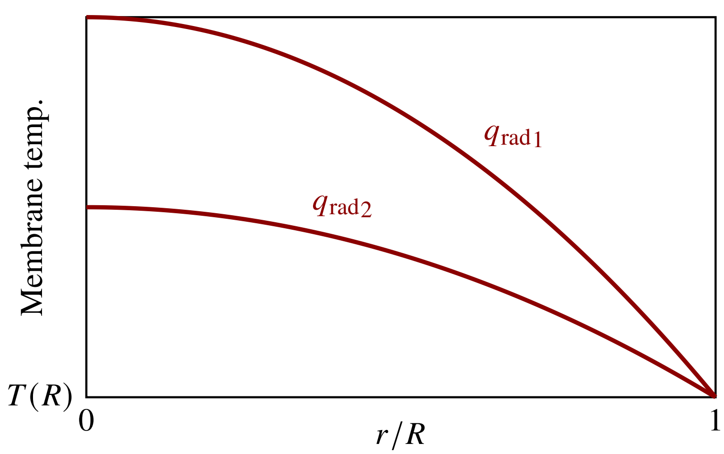
Solve the following problems.

a) For a fixed heat sink temperature, $T(R)$, qualitatively sketch the shape of the temperature distribution in the membrane, $T(r)$, for two heat radiant fluxes $q_{rad\,1}$ and $q_{rad\,2}$, where $q_{rad\,1} > q_{rad\,2}$.

b) Derive the relationship between the radiant heat flux, q_{rad}, and the temperature difference obtained from the thermocouples, ΔT. *Hint:* Treat the absorbed radiant heat flux as if it were a volumetric heat source of magnitude q_{rad}/t W/m^3.

Solution.

a) Since heat flows from the center to the edges, the highest temperature will be at $r = 0$. The slope of the temperature profile must be zero at $r = 0$ by symmetry. The temperatures will be higher when the radiant flux is greater, except at $r = R$ where the temperature is fixed.



b) With the approximations given, the situation can be modeled as one-dimensional, steady heat conduction in cylindrical coordinates, with the absorbed radiation acting like a volumetric heat release. The heat release per unit volume of membrane is q_{rad}/t. With eqns. (2.11) and

(2.13), the heat conduction equation is:

$$\nabla^2 T + \frac{\dot q}{k} = \frac{1}{\alpha}\cancelto{0}{\frac{\partial T}{\partial t}}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 T}{\partial \theta^2}} + \cancelto{0}{\frac{\partial^2 T}{\partial z^2}} = -\frac{\dot q}{k} = -\frac{q_{\text{rad}}}{kt}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{q_{\text{rad}}}{kt}$$

Integrating this o.d.e. twice gives

$$\frac{dT}{dr} = -\frac{q_{\text{rad}} r}{2kt} + \frac{C_1}{r}$$

$$T(r) = -\frac{q_{\text{rad}} r^2}{4kt} + \cancelto{0}{C_1} \ln r + C_2$$

By symmetry, we require that the temperature gradient $dT/dr = 0$ at $r = 0$ (or equivalently, that the temperature be finite at $r = 0$), so $C_1 = 0$.

The temperature difference, $T(0) - T(R)$, follows by subtraction without finding C_2:

$$\boxed{\Delta T = \frac{q_{\text{rad}} R^2}{4kt}}$$

PROBLEM 2.45: You have a 12 oz. (375 mL) can of soda at room temperature (70 °F) that you would like to cool to 45 °F before drinking. You rest the can on its side on the plastic rods of the refrigerator shelf. The can is 2.5 inches in diameter and 5 inches long. The can's emittance is $\varepsilon = 0.4$ and the natural convection heat transfer coefficient around it is a function of the temperature difference between the can and the air: $\overline{h} = 2\,\Delta T^{1/4}$ for ΔT in kelvin.

Assume that thermal interactions with the refrigerator shelf are negligible and that buoyancy currents inside the can will keep the soda well mixed.

a) Estimate how long it will take to cool the can in the refrigerator compartment, which is at 40 °F.
b) Estimate how long it will take to cool the can in the freezer compartment, which is at 5 °F.
c) Are your answers for parts a) and b) the same? If not, what is the main reason that they are different?

SOLUTION. Use a lumped-capacity solution because the liquid in the can is able to circulate, minimizing internal temperature gradients. Treat the soda as having the properties of water (App. A, Table A.3).

Radiation and natural convection act in parallel, just as in Example 2.7, and the effective heat transfer coefficient is the sum of h_{rad} and h_{conv}. Both depend on the temperature difference between the can and the surroundings. While a strict solution would numerically integrate eqn. (1.20) to account for changes in the heat transfer coefficients as the can temperature drops, we will get sufficient accuracy by evaluating h_{rad} and $\overline{h}_{\text{conv}}$ at a single, intermediate temperature difference and applying eqn. (1.22).

The time constant is

$$\boldsymbol{T} = \frac{\rho c V}{A(h_{\text{rad}} + \overline{h}_{\text{conv}})}$$

The volume of the can itself, if calculated, would turn out to be 7% greater than liquid volume. That's because an empty "ullage" space is left to accommodate expansion. For this calculation, we use the true volume of the liquid, since it represents almost all of the mass to be cooled. We'll use the entire surface of the can as heat loss area, however, without trying to account for the ullage.

$$A = \pi D L + 2(\pi D^2/4) = 49.09 \text{ in}^2 = 0.03167 \text{ m}^2$$

$$\rho c V = (999)(4190)(375 \times 10^{-6}) = 1570 \text{ J/kg}$$

a) In this case we begin at 70 °F = 21.11 °C and end at 45 °F = 7.22 °C, with T_∞ = 40 °F = 4.44 °C. If we choose an intermediate can temperature of $(21.11 + 7.22)/2 = 14.17$ °C, we estimate the heat transfer coefficients as

$$\overline{h}_{\text{conv}} = 2\,(14.17 - 4.44)^{1/4} = 3.53 \text{ W/m}^2\text{K}$$

and, with $T_m = (14.17 + 4.44)/2 = 9.31$ °C,

$$h_{\text{rad}} = 4\varepsilon\sigma T_m^3 = 4(0.4)(5.67 \times 10^{-8})(9.31 + 273.15)^3 = 2.04 \text{ W/m}^2\text{K}$$

The time constant is

$$\boldsymbol{T} = \frac{1570}{(0.03167)(3.53 + 2.04)} = 8900 \text{ s}$$

Then, finally,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-t/\boldsymbol{T}}$$

so

$$t_{\text{cool}} = -\boldsymbol{T} \ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = -(8900\text{ s}) \ln\left(\frac{7.22 - 4.44}{21.11 - 4.44}\right)$$

$$= 1.594 \times 10^4 \text{ s} = \underline{4 \text{ hr } 26 \text{ min}}$$

b) Here $T_\infty = 5\ ^\circ\text{F} = -15.00\ ^\circ\text{C}$, so

$$\overline{h}_{\text{conv}} = 2\,(14.17 + 15.00)^{1/4} = 4.65 \text{ W/m}^2\text{K}$$

and, with $T_m = (14.17 - 15.00)/2 = -0.42\ ^\circ\text{C}$,

$$h_{\text{rad}} = 4\varepsilon\sigma T_m^3 = 4(0.4)(5.67 \times 10^{-8})(-0.42 + 273.15)^3 = 1.84 \text{ W/m}^2\text{K}$$

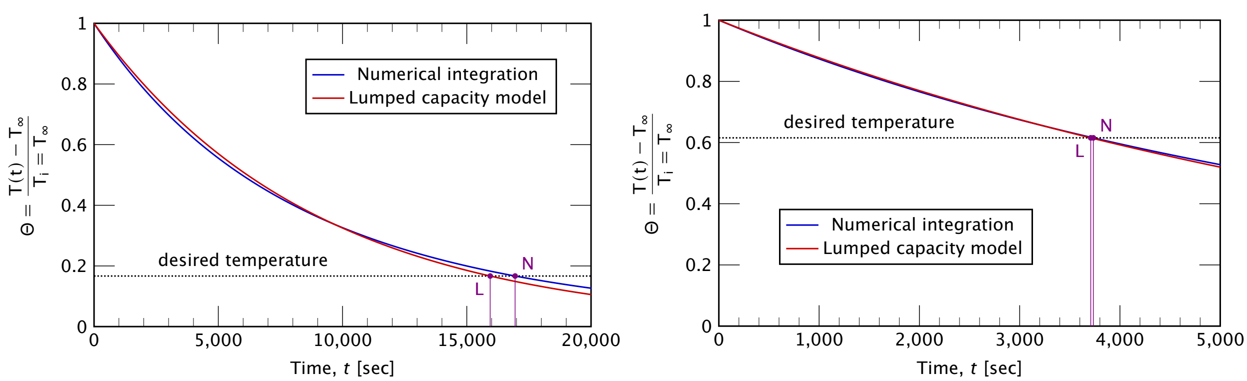
$$\boldsymbol{T} = \frac{1570}{(0.03167)(4.65 + 1.84)} = 7638 \text{ s}$$

$$t_{\text{cool}} = -\boldsymbol{T} \ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = -(7638\text{ s}) \ln\left(\frac{7.22 + 15.00}{21.11 + 15.00}\right)$$

$$= 3709 \text{ s} = \underline{1 \text{ hr } 2 \text{ min}}$$

c) The time to cool in the freezer is less than 1/4 the time to cool in the refrigerator. The reason is that the driving temperature difference for heat transfer is substantially larger throughout the process when cooling in the freezer. The change in the heat transfer coefficients, on the other hand, lowers the time constant by only about 15%. Of course, one must not forget to remove the can from the freezer before the liquid solidifies!

Numerical solutions. Runga-Kutta integrations of eqn. (1.20) are shown in Fig. 1. In the refrigerator, $\overline{h}_{\text{conv}}$ becomes smaller as the can approaches the refrigerator temperature. The numerical solution thus takes about 1000 s longer than the lumped capacity solution (the lumped answer is low by 6%). In the freezer, the heat transfer coefficients do not vary as much, and the lumped result is in good agreement with the numerical solution.



(A) Refrigerator

(B) Freezer

Figure 1. Numerical integration with temperature dependent heat transfer coefficients

Problem 2.46: An exterior wall of a wood-frame house is typically composed, from outside to inside, of a layer of wooden siding, a layer glass fiber insulation, and a layer of gypsum wall board. Standard glass fiber insulation has a thickness of 3.5 inch and a conductivity of 0.038 W/m·K. Gypsum wall board is normally 0.50 inch thick with a conductivity of 0.17 W/m·K, and the siding can be assumed to be 1.0 inch thick with a conductivity of 0.10 W/m·K.

a) Find the overall thermal resistance of such a wall (in K/W) if it has an area of 400 ft^2.
b) The effective heat transfer coefficient (accounting for both convection and radiation) on the outside of the wall is $\overline{h}_o = 20$ W/m^{2}K and that on the inside is $\overline{h}_i = 10$ W/m^{2}K. Determine the total thermal resistance for heat loss from the indoor air to the outdoor air. Also obtain an overall heat transfer coefficient, U, in W/m^{2}K.
c) If the interior temperature is 20°C and the outdoor temperature is −5°C, find the heat loss through the wall in watts and the heat flux in W/m^2.
d) Which of the five thermal resistances is dominant?
e) The wall is held together with vertical wooden studs between the siding and the gypsum. The studs are spruce, 3.5 in. by 1.5 in. on a 16 in. center-to-center spacing. If the wall is 8 ft high, by how much do the studs increase U?

Solution.

a) The wall consists of three thermal resistances in series, each of which is a slab of resistance L/kA. The area is

$$A = 400 \text{ ft}^2 = (400)(0.3048)^2 \text{ m}^2 = 37.16 \text{ m}^2$$

Summing the resistances and converting inches to meters gives

$$\begin{aligned} R_{t_\text{equiv}} &= R_\text{siding} + R_\text{insul} + R_\text{gypsum} \\ &= \frac{(1)(0.0254)}{(0.1)(37.16)} + \frac{(3.5)(0.0254)}{(0.038)(37.16)} + \frac{(0.5)(0.0254)}{(0.17)(37.16)} \\ &= 6.835 \times 10^{-3} + 6.296 \times 10^{-2} + 2.010 \times 10^{-3} \\ &= \underline{0.0718 \text{ K/W}} \end{aligned}$$

b) Now we must add the two convection resistances, $1/\overline{h}A$, in series to R_{t_equiv}:

$$\begin{aligned} R_{t_\text{total}} &= R_\text{conv, out} + R_\text{equiv} + R_\text{conv, in} \\ &= \frac{1}{(20)(37.16)} + 0.0718 + \frac{1}{(10)(37.16)} \\ &= \underline{0.0758 \text{ K/W}} \end{aligned}$$

Next, since $Q = \Delta T / R_{t_\text{total}} = UA\,\Delta T$, we have $UA = 1/R_{t_\text{total}}$. So

$$U = \frac{1}{A\,R_{t_\text{total}}} = \frac{1}{(37.16)(0.0758)} = \underline{0.355 \text{ W/m}^2\text{K}}$$

c)

$$Q = \frac{\Delta T}{R_{t_\text{total}}} = \frac{20 - (-5)}{0.0758} = \underline{330 \text{ W}}$$

$$q = \frac{Q}{A} = \frac{330}{37.16} = \underline{8.88 \text{ W/m}^2}$$

d) The insulation is dominant, contributing to 0.0630 K/W or 83% of the total resistance.

e) An 8 ft high wall of 400 ft^2 area has a length of 50 ft or 600 inches. This allows for 600/16 = 37.5 studs, but the spacing is likely reduced at one end of the wall. Take 38 studs. Each stud has an area of $(1.5/12)(8) = 1$ ft^2, so the total stud area is $A_{\text{studs}} = 38$ ft^2 with the remaining insulated area being $A_{\text{ins}} = 400 - 38 = 362$ ft^2.

We will approximate the configuration as one dimensional heat conduction through parallel resistances, one for the studs and one for the insulation. The total resistance of the insulated portion of the wall is a proportion of that calculated in part b):

$$R_{t_{\text{ins}}} = \frac{400}{362}(0.0758) = 0.0838 \text{ K/W}$$

The total resistance of the stud portion is, with $k_{\text{spruce}} = 0.11$ W/m·K from Appendix A (Table A.2),

$$\begin{aligned} R_{t_{\text{studs}}} &= \frac{1}{A_{\text{studs}}}\left[\frac{1}{20} + \frac{1(0.0254)}{(0.1)} + \frac{3.5(0.0254)}{(0.11)} + \frac{(0.5)(0.0254)}{(0.17)} + \frac{1}{10}\right] \\ &= \frac{1}{38(0.3048)^2}(0.050 + 0.254 + 0.808 + 0.0747 + 0.100) \\ &= 0.365 \text{ K/W} \end{aligned}$$

The parallel resistance is found as in Example 2.7:

$$R_{t_{\text{total}}} = \frac{1}{R_{t_{\text{ins}}}^{-1} + R_{t_{\text{studs}}}^{-1}} = \frac{1}{1/0.0838 + 1/0.365} = 0.0682 \text{ K/W}$$

The studs reduced the total thermal resistance. The overall heat transfer coefficient rises to

$$U_{\text{studs}} = \frac{1}{AR_{t_{\text{total}}}} = \frac{1}{(0.0682)(37.16)} = \underline{0.395 \text{ W/m}^2\text{K}}$$

an increase of 11%.

PROBLEM 2.47: The heat conduction equation in Sect. 2.1 includes a volumetric heat release rate, $\dot{q}$. We normally describe heat as a transfer of energy and entropy across a system boundary, so the notion of volumetric heat release needs some thought. Consider an electrical resistor carrying a current I with a voltage difference of ΔV in steady state. Electrical work is done on the resistor at the rate $\Delta V \cdot I$.

a) Use eqn. (1.1) to find the rate of heat and entropy flow out of the resistor. Assume that the resistor's surface temperature, T, is uniform. What is the rate of entropy generation, $\dot{S}_{\text{gen}}$?
b) Suppose that the resistor dissipates electrical work uniformly within its volume, $\mathcal{V}$, and that its thermal conductivity is high enough to provide a nearly uniform internal temperature. What is the volumetric entropy generation rate, $\dot{s}_{\text{gen}}$?
c) By considering the net heat leaving a differential volume $d\mathcal{V}$, use $\dot{s}_{\text{gen}}$ to define the volumetric heat release rate, $\dot{q}$.
d) If the resistor has a nonuniform internal temperature but a uniform rate of work dissipation, does the total entropy generation change? Why or why not?
e) If the resistor is insulated, so that no heat flows out, what is the entropy generation rate? Assume the resistor's temperature is nearly uniform, starting at T_0 at time $t = 0$.

SOLUTION.

a) The heat flow Q *into* the resistor is related to the work Wk done *by* the resistor

$$Q = Wk + \overbrace{\cancel{\frac{dU}{dt}}}^{0} = -\Delta V \cdot I$$

So, the heat flow *out* of the resistor is $\Delta V \cdot I$. The entropy leaving the resistor is simply

$$\dot{S}_{\text{out}} = -\frac{Q}{T} = \frac{\Delta V \cdot I}{T}$$

Therefore, the rate of entropy generation in the resistor, by dissipation of electrical work, is

$$\dot{S}_{\text{gen}} = \dot{S}_{\text{out}} = \frac{\Delta V \cdot I}{T} \tag{1}$$

b) Dividing eqn. (1) by $\mathcal{V}$, the entropy generated per unit volume is

$$\dot{s}_{\text{gen}} = \frac{\Delta V \cdot I}{\mathcal{V} T}$$

c) The entropy generated in a differential volume $d\mathcal{V}$ must be the net entropy transfer out of that volume. The volume's surface has the local temperature T, so the net heat flow out is

$$dQ_{\text{out}} = T\, d\dot{S}_{\text{out}} = T\, \dot{s}_{\text{gen}}\, d\mathcal{V}$$

The apparent rate of "volumetric heat release" is therefore

$$\dot{q} \equiv T\, \dot{s}_{\text{gen}} = \frac{\Delta V \cdot I}{\mathcal{V}}$$

d) The heat flow out of the surface is unchanged, so the rate of entropy flow out of the resistor is unchanged. Thus, the total rate of entropy generation is unchanged. However, heat transfer from hotter parts of the resistor is associated with lower entropy transfer (since T is greater), but heat conduction through a temperature difference generates additional entropy, cf. eqn. (1.7). The net result is the same overall entropy generation rate.

e) If the resistor is adiabatic, the system is unsteady. With eqn. (1.1),

$$\cancelto{0}{Q} = Wk + \frac{dU}{dt}$$

$$\frac{dU}{dt} = \Delta V \cdot I$$

Assuming an incompressible resistor with a uniform temperature, eqn. (1.4) gives

$$\frac{dS}{dt} = \frac{1}{T(t)}\frac{dU}{dt} = \frac{\Delta V \cdot I}{T(t)}$$

The temperature as a function of time can be found with eqn. (1.3)

$$\frac{dU}{dt} = mc\frac{dT}{dt}$$

and an easy calculation leads to

$$T(t) = T_0 + \frac{\Delta V \cdot I}{mc}\,t$$

Because no entropy was transferred in the heating process, all of the entropy change is by entropy generation, and $\dot{S}_{\text{gen}}$ has the same form as in the steady case.

PROBLEM 2.48: If an overall temperature difference of ΔT is imposed on N thermal resistances in series, show that the temperature difference across the i^{th} thermal resistance is

$$\Delta T_i = \frac{R_i}{\sum_{i=1}^{N} R_i}\,\Delta T$$

SOLUTION. The heat flow through the series of resistors is (cf. Fig. 2.18):

$$Q = \frac{\Delta T}{R_{t_{\text{equiv}}}} = \frac{\Delta T}{\sum_{i=1}^{N} R_i}$$

The same heat flows through the i^{th} resistance:

$$Q = \frac{\Delta T_i}{R_i}$$

Equating these expressions and rearranging leads to the stated result:

$$\frac{\Delta T_i}{R_i} = \frac{\Delta T}{\sum_{i=1}^{N} R_i}$$

$$\boxed{\Delta T_i = \frac{R_i}{\sum_{i=1}^{N} R_i}\,\Delta T}$$

In electric circuit theory, this is called the *voltage divider* relationship.

PROBLEM 2.49: An electrical resistor is a 1 mm thick annulus of Inconel (Fig. 1). It dissipates 9.4 kW/m. The resistor is insulated on both sides by a 3 mm layer of epoxy ($k_e = 0.5$ W/m·K). A 316 stainless steel pipe inside the resistor is cooled internally by flowing water. The pipe is 5 cm I.D. and 6 cm O.D. A larger pipe forms an annular passage outside the resistor, through which water also flows; $\overline{h}_{\text{inside}} = \overline{h}_{\text{outside}} = 1400$ W/m^{2}K. The outer pipe has 8.7 cm I.D. and a 0.5 cm wall thickness and is wrapped with 2 cm thick glass-fiber pipe insulation, surrounded outside by ambient air. If the water temperature inside is 47 °C and that outside is 53 °C, find the resistor's temperature.

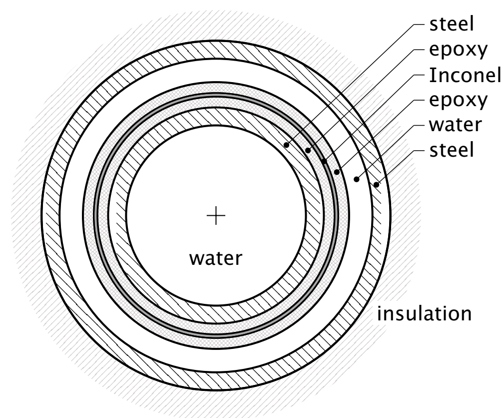


FIGURE 1. Cross-section of resistor with water cooling

SOLUTION. This problem can be solved with two effective resistances, one from the resistor to the water inside and one from the resistor to the water outside. (The insulation and outer pipe can be ignored because the outside water temperature is known.) Further, the epoxy thickness is much smaller than the radius, so it may be treated as a slab (Prob. 2.43).

The internal resistances, in series, may be summed for a 1 m length

$$\begin{aligned}
R_{\text{inside}} &= R_{\text{epoxy}} + R_{\text{pipe}} + R_{\text{conv}} \\
&= \frac{t}{[2\pi(r_o + \delta/2)l]k_e} + \frac{\ln(r_o/r_i)}{2\pi k_{ss} l} + \frac{1}{(2\pi r_i l)\overline{h}_{\text{inside}}} \\
&= \frac{0.003}{2\pi(0.0315)(0.5)} + \frac{\ln(0.03/0.025)}{2\pi(14)} + \frac{1}{2\pi(0.025)(1400)} \\
&= 3.03\times10^{-2} + 2.07\times10^{-3} + 4.55\times10^{-3} = 3.69\times10^{-2} \text{ K/W}
\end{aligned}$$

The epoxy is clearly the dominant resistance. The exterior resistance is

$$\begin{aligned}
R_{\text{outside}} &= R_{\text{epoxy}} + R_{\text{conv}} \\
&= \frac{0.003}{2\pi(0.0355)(0.5)} + \frac{1}{2\pi(0.037)(1400)} \\
&= 2.69\times10^{-2} + 3.07\times10^{-3} = 3.00\times10^{-2} \text{ K/W}
\end{aligned}$$

Heat leaving the resistor goes into both effective resistances. With $Q = 9.4$ kW,

$$Q = \frac{T_{\text{resistor}} - T_{\text{water, inside}}}{R_{\text{inside}}} + \frac{T_{\text{resistor}} - T_{\text{water, outside}}}{R_{\text{outside}}}$$

$$T_{\text{resistor}}\left(\frac{1}{R_{\text{inside}}} + \frac{1}{R_{\text{outside}}}\right) = \left(\frac{T_{\text{water, inside}}}{R_{\text{inside}}} + \frac{T_{\text{water, outside}}}{R_{\text{outside}}}\right) + Q$$

$$T_{\text{resistor}}\underbrace{\left(\frac{1}{3.69\times10^{-2}} + \frac{1}{3.00\times10^{-2}}\right)}_{64.28} = \underbrace{\left(\frac{47}{3.69\times10^{-2}} + \frac{53}{3.00\times10^{-2}}\right)}_{3040} + 9400$$

Solving, $T_{\text{resistor}} = \underline{194\ ^\circ\text{C}}$