A Heat Transfer Textbook

Fifth Edition

Solutions Manual for Chapter 3

by

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3.1 Can both flows be mixed in a cross-flow heat exchanger.

The fluids might be permitted direct contact, of course, but we would have to separate them at the exit. However, it appears that any walls separating the fluids would either eliminate mixing in one flow or restrict it in both flows. An exception to this would be crossflow over a <u>single</u> dividing plate -- not a very effective exchanger, but a legitimate instance of both flows being mixed.

3.2 Find the appropriate mean radius for heat transfer through a single spherical shell, such that $Q = kA(\bar{r})\dfrac{\Delta T}{r_o - r_i}$.

general solution: $T = C_1 + \dfrac{C_2}{r}$ b.c.'s $T(r_i) = T_i$; $T(r_o) = T_o$

so: $T_i = C_1 + \dfrac{C_2}{r_i}$

$T_o = C_1 + \dfrac{C_2}{r_o}$

$T_i - T_o = \Delta T = C_2\left(\dfrac{1}{r_i} - \dfrac{1}{r_o}\right)$

so $C_2 = \dfrac{r_i r_o \Delta T}{r_o - r_i}$

Then $C_1 = T_i - \dfrac{r_o \Delta T}{r_o - r_i}$

and $T = T_i - \dfrac{r_o \Delta T}{r_o - r_i}\left[1 - \dfrac{r_i}{r}\right]$

Finally $Q = -k\,4\pi r^2\left[-\dfrac{r_o r_i \Delta T}{r^2(r_o - r_i)}\right] = 4\pi k \Delta T \dfrac{r_o r_i}{r_o - r_i} = \dfrac{\Delta T}{\dfrac{r_o - r_i}{4\pi k r_o r_i}}$

If we compare $R_{t_{cond}}$ for the two Q equations:

$\dfrac{r_o - r_i}{kA(\bar{r})} = \dfrac{r_o - r_i}{4\pi k r_o r_i}$; $A(\bar{r}) = 4\pi r_o r_i$ so $\underline{\underline{\bar{r} = \sqrt{r_o r_i}}}$

this is a <u>GEOMETRIC</u> mean

3.3 Rework Problem 2.14 using the methods of Chapter 3.

$$UA(LMTD) = 300(2)\,\frac{100 - (120 - T_e)}{\ln \dfrac{100}{120 - T_e}} = \underbrace{\dot{m}c_p}_{503}\,(T_e - 20)$$

so $\quad T_e = 20 + 1.193\,(T_e - 20)\Big/\ln\dfrac{100}{120 - T_e}$

Solve this by trial and error:

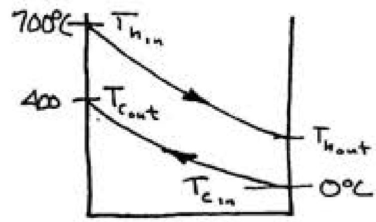
T_e	RHS
80	98.1
85	93.1
90	89.4
89.7	89.64

$\therefore\ T_e \cong 89.7\,^{\circ}C$

This is the same result as we got in Problem 2.14.

3.4 Consider counterflow heat exchanger shown with

$$C_c = C_h = 2.4\,\frac{kg}{s}\;800\,\frac{J}{kg\text{-}K} = 2(960) = \underline{1920\,\frac{J}{s\,^{\circ}C}}$$



$$T_{h_{out}} = T_{h_{in}} - \frac{C_c}{C_h}\left(T_{c_{out}} - T_{c_{in}}\right) = 700 - 400 = \underline{300^{\circ}C}$$

In this case $\overline{\Delta T} = \Delta T_a = \Delta T_b = 300^{\circ}C$.

But if $T_{c_{out}}$ were changed to 500°C, $T_{h_{out}}$ would be 200°C and so too would be $\overline{\Delta T}$. Then

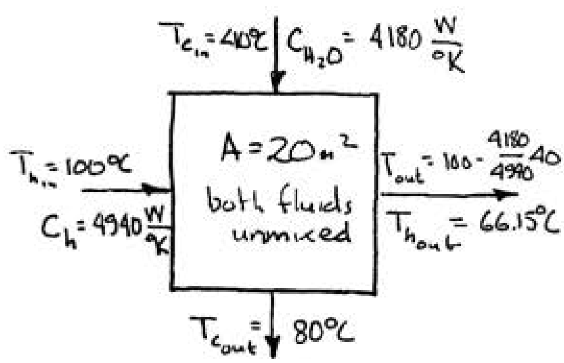
$$C_c(400) = U(A\overline{\Delta T})_{before} \; ; \quad C_c(500) = U(A\overline{\Delta T})_{after}$$

and we would get

$$\frac{C_c}{U} = \frac{A_{before}\,300}{400} = \frac{A_{after}\,200}{500} \; ; \quad \frac{A_a}{A_b} = \frac{300(500)}{200(400)} = \frac{15}{8}$$

Then $\quad \dfrac{A_{after} - A_{before}}{A_{before}} = \dfrac{15}{8} - 1 = \underline{\underline{87.5\%}}$

3.5 Find U for the exchanger shown



effectiveness method

$$Q = 4180(40) = \epsilon\,\underbrace{C_{min}}_{4180}\,\underbrace{(T_{h_{in}} - T_{c_{in}})}_{60}$$

so $\quad \epsilon = \frac{2}{3} \quad$ where $\dfrac{C_{min}}{C_{max}} = 0.846$

then from Fig. 3.17a we read $NTU = \dfrac{UA}{C_{min}} = 2.45$.

3.5 Continued:

$$\text{Therefore } \bar{U} = \frac{2.45(4180)}{20} = \underline{\underline{512 \frac{W}{m^2\text{-}^\circ C}}}$$

LMTD method

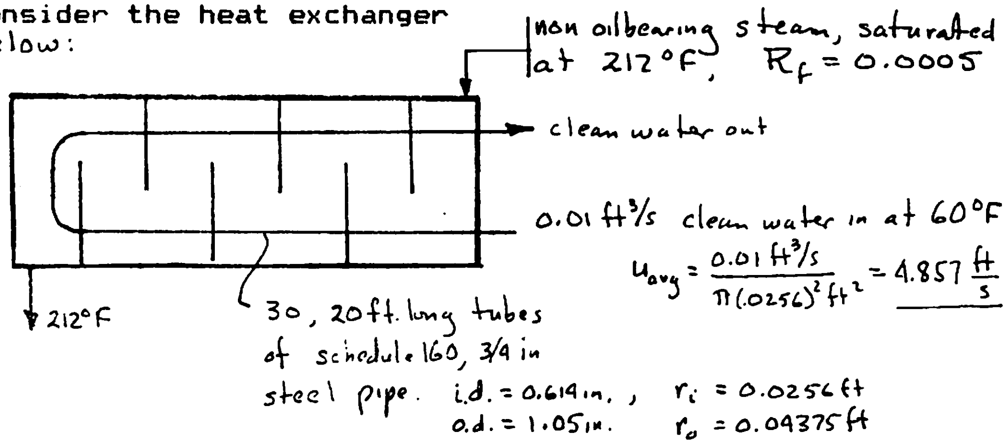
$$P = \frac{80-40}{100-40} = \frac{2}{3}\,;\quad R = \frac{100-66.15}{40} = 0.846\,;\quad \text{from Fig. 3.14c},\ F = 0.805$$

$$LMTD = [(100-80)-(66.15-40)]/\ln\frac{20}{26.15} = 22.94^\circ C\,;\quad \bar{U} = \frac{Q}{AF\,LMTD} = \frac{4180(40)}{20(0.805)(22.94)}$$

$$= \underline{\underline{453 \frac{W}{m^2\text{-}^\circ C}}}$$

(There is a 12 % or so disagreement in graph reading.)

3.6 Consider the heat exchanger below:



$$u_{avg} = \frac{0.01\ ft^3/s}{\pi(.0256)^2\ ft^2} = \underline{4.857\ \frac{ft}{s}}$$

Take $\bar{h}$ inside the tubes to be 1380 Btu/ft^2-hr-$^\circ$F and $\bar{h}_{cond}$ as 2000 Btu/ft^2-hr-$^\circ$F.

Find $T_{water,out}$ and $\dot{m}_{condensate}$. To do this first find U based on the outside tube area.

$$U = \left[\frac{r_o}{r_i h_i} + \frac{r_o \ln(r_o/r_i)}{k_{steel}} + \frac{1}{h_{cond}} + R_f\right]^{-1}$$

$$= \left[\frac{0.04375}{0.0256(1380)} + \frac{0.04375}{26}\ln\frac{0.04375}{0.0256} + \frac{1}{2000} + 0.0005\right]^{-1}$$

$$= \underline{318.4\ Btu/ft^2\text{-}hr\text{-}^\circ F}$$

Next look up the effectiveness, ϵ, in Fig. 3.16 or 3.17 for $C_{min}/C_{max} = 0$ and for

$$NTU = \frac{UA}{C_{min}} \quad \frac{318.4[30(2\times 20)\pi(2\times 0.04375)]}{30(0.01)(\rho = 62.9)(3600)} = 1.558$$

on any of the 6 curves we read $\underline{\epsilon \simeq 0.795}$

Then $$Q = \epsilon C_{min}(T_{h_{in}} - T_{c_{out}}) = 0.795(30)(0.01)(6.24)(3600)(212-60)$$

$$= \underline{8.14\times 10^6\ \frac{Btu}{hr}}$$

3.6 (Continued)

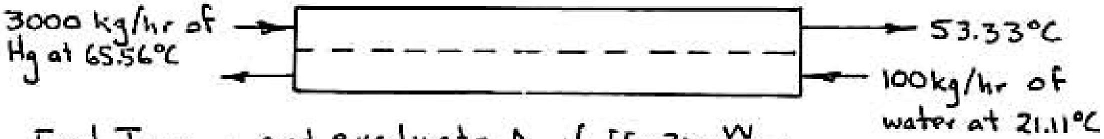
and $\dot{m}_{condensate} = \frac{Q}{h_{fg}} = \frac{8.14 \times 10^6}{970.3} = \underline{\underline{8393 \ \frac{lb_m}{hr.}}}$

Finally: $\epsilon = 0.795 = \frac{T_{cond} - T_{c_{in}}}{212-60}$;

so $T_{c_{out}} = (212-60)(0.795) + 60 = \underline{\underline{181\ ^\circ F}}$

[Note that, had we taken <u>each</u> U-tube to be 20 ft long, the numbers above would have come out as follows: NTU = 0.779, ϵ = 0.55, Q = 5.63 x 10^6, m_{cond} = 5810, and $T_{c_{out}}$ = 144.]

3.7 Consider a counter-flow heat exchanger as shown:



Find $T_{H_2O_{out}}$ and evaluate A, if $U = 300 \frac{W}{m^2\text{-}^\circ C}$.

$$(\dot{m}c_p)_{H_2O}(T_{H_2O_o} - 21.11) = (\dot{m}c_p)_{Hg}(65.56-53.33)$$

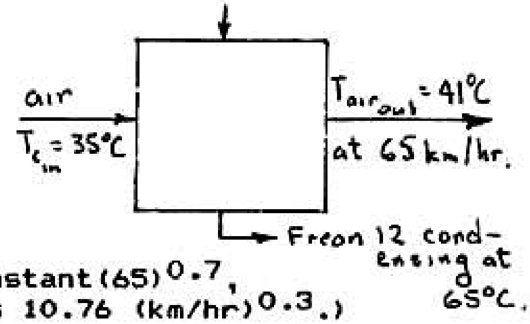
so: $T_{H_2O_{out}} = 21.11 + \frac{3000(139.4)}{100\ (4180)}(65.56-53.33) = \underline{\underline{33.35\ ^\circ C}}$

The temperature changes of the two streams are almost equal -- 32.22 K and 32.21 K. So this is a balanced counterflow heat exchanger (see Example 3.2) whose effective LMTD we can call 32.215 K. These temperature differences are so similar because C_h and C_c are nearly the same.

And $Q = (\dot{m}c_p)_{Hg}(65.56-53.33) = \frac{5.112(10)^6}{3600} = UA(LMTD)$

so: $A = \frac{1420.8}{300\ (32.215)} = \underline{\underline{0.1470 m^2}}$

3.8 An automobile heat exchanger is of cross-flow type with both fluids unmixed as shown:



Q = 18 kW at 65 km/hr and

U = constant (velocity)$^{0.7}$

(and, since U = 200km/hr = constant(65)$^{0.7}$, the constant must come out as 10.76 (km/hr)$^{0.3}$.)

Plot the reduction of Q <u>vs.</u> the speed of the auto.

This is an effectiveness problem and we could use the charts, but it is simpler to use equation (3.20) or (3.21) with C_{min}/C_{max} = 0: Thus ϵ =1 - exp(-NTU). To use this we

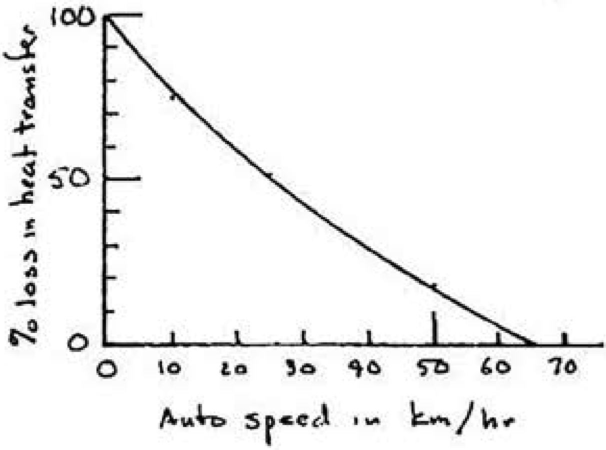
3.8 (Continued)

must first calculate the area of the exchanger. At design conditions, $\epsilon = 6/(65 - 35) = 0.20$, so NTU = 0.223 or

$$A = 0.223C_{min}/U = 0.223\ \frac{18{,}000/6}{200} = \underline{3.34\ m^2}$$

and <u>that</u> will not change. Now we use a tabular solution.

vel km/hr	$U\ \frac{W}{m^2K} = 10.76\,Vel^{0.7}$	$NTU = \frac{A}{C_{min}}U = 0.07222U/Vel$	$\epsilon = 1 - e^{-NTU}$	$T_{air\ out} = 35+(65-35)\epsilon$	$Q = C_{min}\frac{Vel}{65}\Delta T = 46.2\,Vel(T_{out}-35)$	% reduction in Q
65	200	0.223	0.200	41.0	18,000 W	0
50	166	0.240	0.213	41.4	14,768	18.0
25	102	0.294	0.255	42.65	8,826	51.0
10	54	0.390	0.323	44.7	4,477	75.1



3.9 Derive the effectiveness for a counterflow heat exchanger: Equation (3.9b) for the counterflow case reduces to

$$\ln \frac{\overbrace{(T_{h_i} - T_{c_c}) - (T_{c_o} - T_{c_i})}^{\Delta T_a}}{(-1+\frac{C_c}{C_h})(T_{c_i} - T_{c_o}) + \Delta T_a} = \ln \frac{1}{(-1+\frac{C_c}{C_h})\dfrac{1}{-\frac{1}{\epsilon}\frac{C_c}{C_{min}}+1} + 1} = \ln \frac{-\frac{1}{\epsilon}\frac{C_c}{C_{min}} + 1}{-1+\frac{C_c}{C_h} - \frac{1}{\epsilon}\frac{C_c}{C_{min}} + 1}$$

$$= \pm \ln \frac{-\frac{1}{\epsilon} + 1}{\frac{C_{min}}{C_{max}} - \frac{1}{\epsilon}} = \mp NTU\left[1 - \frac{C_{min}}{C_{max}}\right]$$ where upper sign is for $C_c = C_{min}$; lower sign is for $C_h = C_{min}$.

We can then drop lower sign & solve for ϵ:

$$-\frac{1-\epsilon}{\frac{C_{min}}{C_{max}}\epsilon - 1} = e^{-NTU\left[1 - \frac{C_{min}}{C_{max}}\right]} \quad \text{or} \quad \epsilon = \underline{\underline{\frac{1 - \exp\left[-(1 - \frac{C_{min}}{C_{max}})NTU\right]}{1 - \frac{C_{min}}{C_{max}}\exp\left[1-(1-\frac{C_{min}}{C_{max}})NTU\right]}}}$$

3.10 Find the limiting effectiveness for the parallel and counterflow cases:

If NTU $\to \infty$ the outlet temperatures for the //-flow case -- equal.

Then $\Delta T_{hot} + \Delta T_{cold} = \Delta T_{overall}$ and:

$$\varepsilon \equiv \frac{C_h}{C_{min}} \frac{\Delta T_h}{\Delta T_{overall}} \quad \text{while} \quad \frac{\Delta T_h}{\Delta T_{overall}} = \frac{1}{1+\dfrac{\Delta T_c}{\Delta T_h}} = \frac{1}{1+\dfrac{C_h}{C_c}}$$

Thus:

$$\underline{\underline{\varepsilon = \frac{C_h}{C_{min}} \left(\frac{C_c}{C_c+C_h} \right) = \frac{1}{1+\dfrac{C_{min}}{C_{max}}}}} \quad \text{(for either } C_c \text{ or } C_h = C_{min}\text{.)}$$

This compares correctly with Fig. 3.16 or eqn. (3.20)

In the counterflow case, the maximum possible heat transfer, $C_{min}(T_{h_{in}} - T_{c_{in}})$, will be transferred to the minimum heat capacity flow if the heat exchanger is made long enough that NTU approaches infinity. In this case

$$\underline{\underline{\varepsilon = 1}}$$

This is borne out in the trend of Fig. 3.16 and by eqn. (3.21) as NTU approaches infinity.

3.11 Derive $\varepsilon = \varepsilon(NTU, C_{min}/C_{max})$ for a heat exchanger in which one flow is isothermal.

If the hot flow is isothermal, $C_h = \infty$, and either of eqns. (3.9) gives

$$\ln\left[-\frac{T_{c_o}-T_{c_i}}{T_h-T_{c_i}}+1\right] = \ln\left[1-\frac{\Delta T_c}{T_h-T_{c_i}-\Delta T_c}\right] = \ln\left[1-\frac{1}{\frac{1}{\varepsilon}-1}\right] = \ln\frac{1}{1-\varepsilon} = -\frac{UA}{C_{min}} \equiv -NTU$$

Therefore

$$1-\epsilon = e^{-NTU} \quad \text{or} \quad \underline{\underline{\varepsilon = 1-e^{-NTU}}}$$

Notice that this is also the limit of both eqns. (3.20) & (3.21) where $C_{min}/C_{max} \to 0$. It is also true if $C_c = \infty$.

3.12 A single pass heat exchanger condenses steam at 100°C on the shell side and heats water from 10 to 30°C on the other side. $U = 2500$ W/m-°C and $A = \pi(0.05\text{ m})(2\text{ m}) = 0.3142\text{ m}^2$.

a) Should it be //-or counter-flow? It doesn't matter since T_h = const.

b) Find the LMTD

$$LMTD = \frac{(100-10)-(100-30)}{\ln\frac{90}{70}} = \underline{\underline{79.58°C}}$$

c) Find $\dot{m}_{H_2O}$

$$\dot{m}_{H_2O} = \frac{UA(LMTD)}{c_p\Delta T} = \frac{2500(0.314)(79.6)}{4180(20)} = \underline{\underline{0.747\ \frac{kg}{s}}}$$

$$\dot{m}_{steam} = \frac{Q}{h_f} = \frac{2500(0.314)(79.6)}{2257000} = \underline{\underline{0.277\ \frac{kg}{s}}}$$

d) Find ε:

$$\varepsilon = \frac{Q}{\dot{m}c_p\Delta T_{overall}} = \frac{2500(0.314)(79.6)}{0.747(4180)(100-10)} = \underline{\underline{0.222}}$$

3.13 Two kg/s of air at 27°C and 1.5 kg/s of water at 60°C enter a heat exchanger. $U = 185\ \mathrm{W/m^2\text{-}°C}$ and $A = 0.12\ \mathrm{m^2}$. Find T_{h_o} & T_{c_o} if

a) The exchanger is //-flow: $C_c = C_{air} = 2(1006) = 2012$, $C_h = C_{H_2O} = 1.5(4184) = 6276$ $\Bigg\}\ \dfrac{C_{min}}{C_{max}} = 0.321$

$$NTU = UA/C_{min} = 185(12)/2012 = 1.103$$

$$\text{From Fig. 3.16, } \varepsilon = 0.57 = \frac{C_{min}}{C_{min}}\left(\frac{T_{c_o} - 27}{60-27}\right); \quad T_{c_{out}} = \underline{\underline{45.8°C}}$$

$$\text{and } T_{h_o} = 60 - \frac{2012(45.8-27)}{6276} = \underline{\underline{54.0°C}}$$

b) The exchanger is counter-flow: (C_c, C_h, NTU are the same)

$$\text{From Fig. 3.16, } \varepsilon = 0.57 = \frac{C_{min}}{C_{min}}\left(\frac{T_{c_{out}} - 27}{60-27}\right); \quad T_{c_{out}} = \underline{\underline{45.8°C}}$$

$$\text{and } T_{h_o} = 60 - \frac{2012(45.8-27)}{6276} = \underline{\underline{53.97°C}}$$

c) The exchanger is crossflow with one stream mixed:

$$\text{From Fig. 3.17b), } \varepsilon = 0.615 = \frac{C_{min}}{C_{min}}\left(\frac{T_{c_{out}} - 27}{60-27}\right); \quad T_{c_{out}} = \underline{\underline{47.3°C}}$$

$$\text{and } T_{h_o} = 60 - \frac{2012(47.3-27)}{6276} = \underline{\underline{53.5°C}}$$

d) The exchanger is crossflow with both streams unmixed

$$\text{From Fig. 3.17a), } \varepsilon = 0.6 = \frac{C_{min}}{C_{min}}\left(\frac{T_{c_{out}} - 27}{60-27}\right); \quad T_{c_{out}} = \underline{\underline{46.8°C}}$$

$$\text{and } T_{h_o} = 60 - \frac{2012(46.8-27)}{6276} = \underline{\underline{53.65°C}}$$

3.14 0.25 kg/s of air at 0^oC enters a cross-flow exchanger (C_c = 251 $W/^oC$.) It is to be heated to 20^oC by 0.14 kg/s of air at 50^oC (C_h = 140 $W/^oC$.) The streams are unmixed. Plot U vs. A and identify a reasonable operating range.

$$Q = C_c(20 - 0) = 5020 \text{ W} = C_c(50 - T_{h_o}); \quad T_{h_o} = 50 - \frac{5020}{140} = 14.14^oC$$

Using Fig. 3.14c and eqn. (3.15):

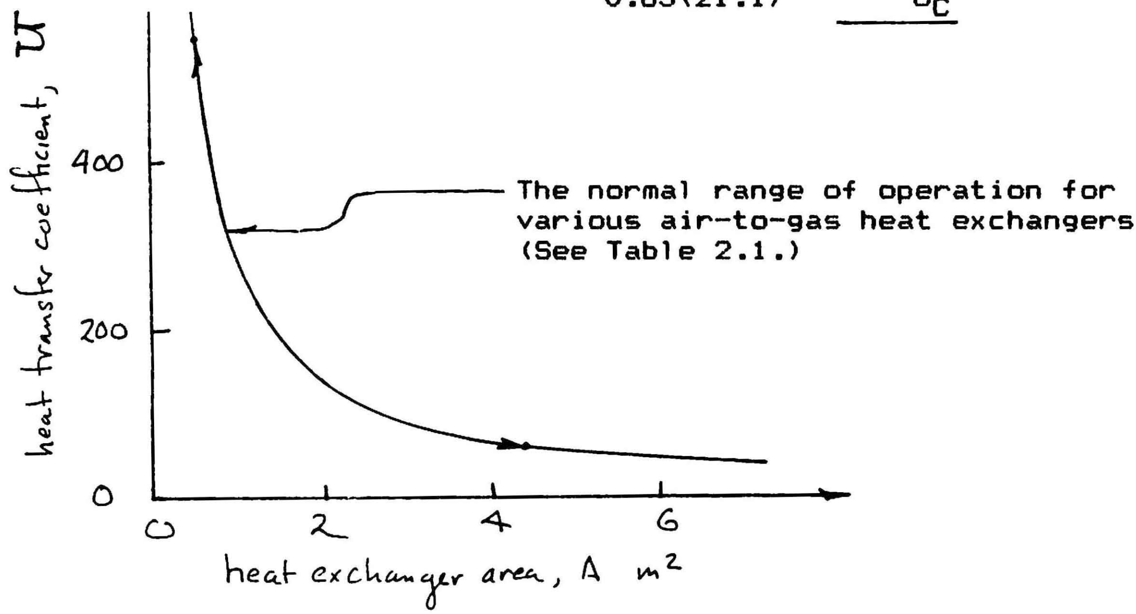
$$F(PR, \frac{1}{R}) = F[0.4(1.79), \frac{1}{1.79}]$$

$$= F(0.716, 0.559) = 0.85$$

$$\text{and LMTD} = \frac{(50 - 20) - (14.14 - 0)}{\ln(30/14.14)} = 21.1^oC$$

Then:

$$q = (UA)F(LMTD) \quad \text{or} \quad UA = \frac{5020}{0.85(21.1)} = \underline{280 \; \frac{W}{^oC}}$$



The area should be between 0.6 and 4 m^2 -- say 2 m^2.

3.15 A two-shell pass, 4-tube pass heat exchanger uses 20 kg/s of water at 10 °C to cool 8 kg/s of processed water from 80°C to 20°C on the tube side. $U = 800\ \mathrm{W/m^2\text{-}°C}$. Find $T_{c_{out}}$ and A.

$$C_h/C_c = 8/20 = 0.4 = C_{min}/C_{max}$$

Then:
$$T_{c_{out}} = \frac{C_h}{C_c}(T_{h_{in}} - T_{h_{out}}) + T_{c_{in}} = 0.4(55) + 10 = \underline{\underline{32°C}}$$

and:
$$A = \frac{(\dot m c_p)_{cold}\,(22)}{U\,(F \cdot \mathrm{LMTD})}$$

From Fig. 3.14b we read $\underline{F = 0.93}$ based on $R = 0.4$ and $P = 0.7857$

so:
$$A = \frac{20(4180)(22)}{800\left(0.93\,\dfrac{(80-32)-(25-10)}{\ln(38/15)}\right)} = \underline{\underline{87.1\ m^2}}$$

3.16 $U = 2000$ for a new exchanger. $T_{c_{in}} = 25°C$, $T_{c_o} = 80°C$, $T_{h_i} = 160°C$, & $T_{h_o} = 70°C$. After 6 mo., $T_{h_o} = 90°C$ and Q is reduced by 30%. What is R_f now?

For the new exchanger:
$$\mathrm{LMTD} = \frac{(160-80)-(70-25)}{\ln 80/45} = \underline{60.83°C}$$

and from Fig 3.14d:
$$\frac{T_{t_o} - T_{t_i}}{T_{s_i} - T_{t_i}} = \frac{70-160}{25-160} = \frac{2}{3}\ ; \quad \frac{T_{s_i} - T_{s_o}}{T_{t_o} - T_{t_i}} = \frac{25-80}{70-160} = 0.611, \text{ so } \underline{F = 0.75}$$

Notice that fouling must reduce the hot side flow rate by 10% as well as increasing outlet temp. of hot flow. If flow rate of cold water stays the same, then the new $T_{c_{out}}$ must be $\underline{63.5°C}$.

For the used exchanger:
$$\mathrm{LMTD} = \frac{(160-63.5)-(90-25)}{\ln(96.5/65)} = \underline{79.7°C}$$

and from Fig. 3.14d:
$$\frac{T_{t_o} - T_{t_i}}{T_{s_i} - T_{t_i}} = \frac{90-160}{25-160} = 0.5185 \ \ \& \ \ \frac{T_{s_i} - T_{s_o}}{T_{t_o} - T_{t_i}} = \frac{25-63.5}{90-160} = 0.55$$

so: $\underline{F = 0.92}$

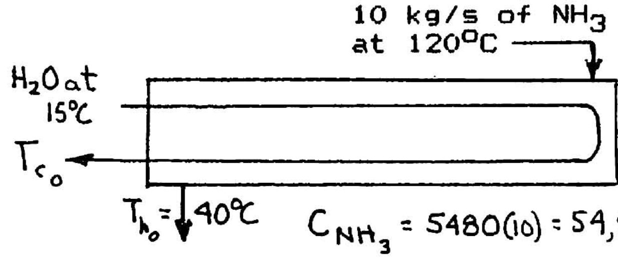
Then:
$$\frac{Q_{new} - Q_{old}}{Q_{new}} = 0.3 = \frac{(U\cancel{A}F \cdot \mathrm{LMTD})_{new} - (U\cancel{A}F \cdot \mathrm{LMTD})_{old}}{(U\cancel{A}F \cdot \mathrm{LMTD})_{new}}$$

$$0.3 = \frac{2000(0.75)(60.83) - U_{old}(0.92)(79.7)}{2000(0.75)(60.83)}$$

From which we obtain: $\underline{U_{old} = 871\ \mathrm{W/m^2\text{-}°C}}$

And:
$$R_f = \frac{1}{U_{old}} - \frac{1}{U_{new}} = \frac{1}{871} - \frac{1}{2000} = \underline{\underline{0.00065\ \frac{m^2\text{-}°C}{W}}}$$

3.17 Determine the mass flow rate of condensate for the heat exchanger shown:



$C_{NH_3} = 5480(10) = 54{,}800 \frac{W}{°C}$

We do not know whether C_{NH_3} is C_{max} or C_{min}. Let's see what happens if it is C_{min}.

$$\frac{UA}{C_{min}} = \frac{1500(90)}{54{,}800} = 2.46$$

$$\epsilon = \frac{C_h}{C_h}\frac{120-40}{120-15} = 0.762$$

From Fig. 3.17c we read: $\frac{C_{min}}{C_{max}} = 0.35$

So the assumption was correct.

Then:

$$C_{max} = C_{H_2O} = \frac{54{,}800}{0.35} = 1.57 \times 10^5 \ \frac{W}{°C}$$

This gives:

$$Q = C_{NH_3}(120-40) = C_{H_2O}(T_{c_o} - 15)$$

$$T_{c_o} = 15 + \frac{54{,}800}{157{,}000}(80) = \underline{43°C}$$

and

$$\dot m_{H_2O} = \frac{C_{H_2O}}{c_{p\,H_2O}} = \frac{157{,}000}{4177} = \underline{\underline{37.6 \ \frac{kg}{s}}}$$

Finally:

$$Q = 54{,}800(80) = 4{,}384{,}000 \text{ W} = \underline{4{,}384 \text{ kW}}$$

3.18 Find the exit temperatures for the heat exchanger in Example 3.5 if the configuration is changed to 2 shell pass, 4 tube passes. What area would give the same value of $T_{h_{out}}$.

We still have NTU = 1.5 and $C_{min}/C_{max} \simeq 0.5$, so Fig. 3.17d gives $\epsilon = 0.67$. Then

$$Q = 0.67(10{,}000)(110) = 737{,}000 \text{ W},$$

and we get

$$T_{h_o} = 150 - 737{,}000/10{,}000 = \underline{\underline{76.3\ °C}}$$

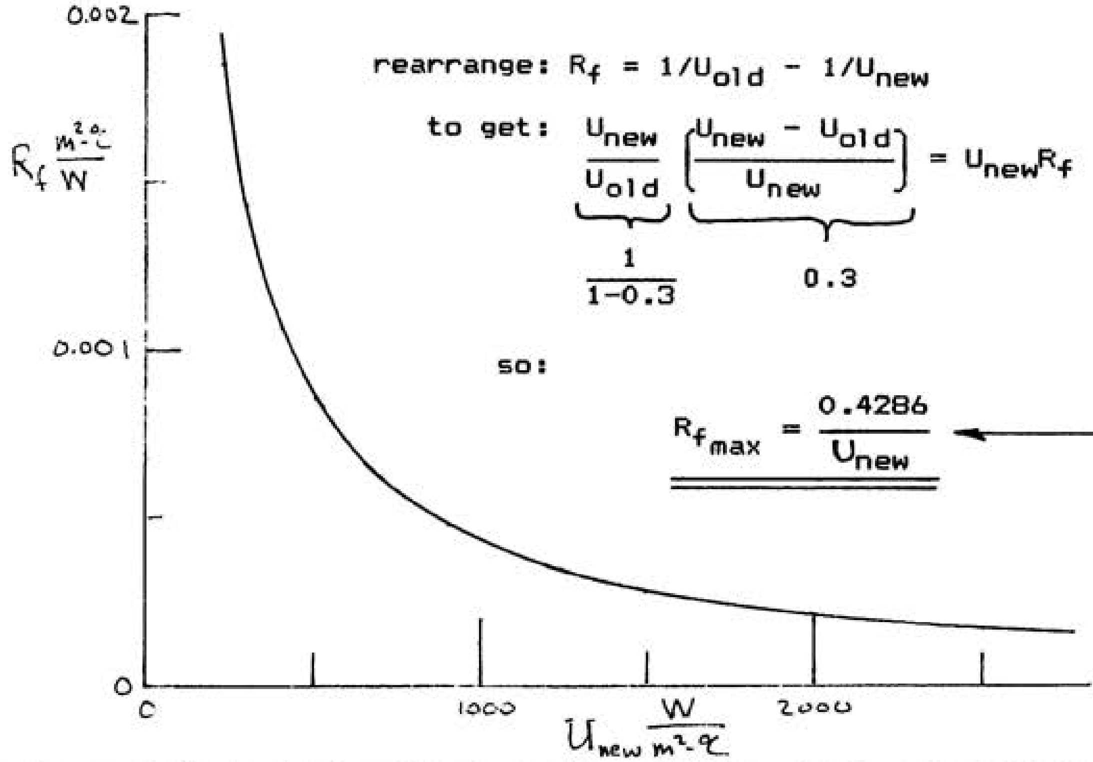
$$T_{c_o} = 40 + 737{,}000/20{,}000 = \underline{\underline{76.85°C}}$$

To get T = 84.44°C, calculate

$$\epsilon = \frac{150 - 84.44}{150 - 40} = 0.596$$

so Fig. 3.17d gives NTU = 1.1 = 0.05A. Then A = $\underline{\underline{22.0 \text{ m}^2}}$

3.19 Plot R_f vs. U_{new} if a 30 % reduction in U is the maximum tolerable.



rearrange: $R_f = 1/U_{old} - 1/U_{new}$

to get: $$\underbrace{\frac{U_{new}}{U_{old}}}_{\frac{1}{1-0.3}} \underbrace{\left[\frac{U_{new} - U_{old}}{U_{new}}\right]}_{0.3} = U_{new}R_f$$

so:

$$\underline{\underline{R_{f_{max}} = \frac{0.4286}{U_{new}}}}$$

3.20 0.8 kg/s of water enters the tubes of a 2 shell-pass, 4 tube-pass heat exchanger, at 17°C and leaves at 37°C. It cools 0.5 kg/s of air entering the shell at 250°C. U=432 W/m^2-°C. Determine: a) the exit air temperature; b) the area of the heat exchanger; and c) the exit temperatures if, after some time, the tubes become fouled with R_f = 0.0005 m^2-°C/W.

a) $C_{H_2O} = 0.8(4177) = 3342$ J/s-°C , $C_{air} = 0.5(1029) = 514.5$ J/s-°C

then $Q = C_{H_2O}\Delta T_{H_2O} = 3342(20) = 66{,}840$ J/s $= C_{air}\Delta T_{air} = 514.5(250 - T_{air_{out}})$

$$\therefore \; \underline{\underline{T_{air_{out}} = 120.1\,°C}}$$

b) with reference to Fig. 3.13: $$LMTD = \frac{(250-37)-(120-17)}{\ln 213/103} = \underline{151.4°C}$$

and $P = \frac{37-17}{250-17} = 0.086$; $R = \frac{250-120.1}{20} = 6.5$, so $F_{Fig.\,3.14b}\left(0.086(6.5), \frac{1}{6.5}\right)$
$= F(0.559, 0.154) \simeq 1$

Then: $$A = \frac{Q}{U(LMTD)F} = \frac{66{,}840}{432(151.4)(1.0)} = \underline{\underline{1.022\ m^2}}$$

3.20 (continued)

c.) First we must calculate the fouled value of U:

$$\frac{1}{U_{fouled}} = \frac{1}{U_{new}} + R_f = \frac{1}{432} + 0.0005; \quad \underline{U_f = 355.3 \frac{W}{m^2\text{-}^\circ C}}$$

We find the exit temperatures using the effectiveness method:

$$\frac{C_{min}}{C_{max}} = \frac{514.5}{3342} = \underline{0.154} \quad \& \quad NTU = \frac{355.3(1.022)}{514.5} = \underline{0.706}$$

so from Fig. 3.17d we read: $\epsilon = 0.49 = \frac{C_{H_2O}(T_{H_2O_{out}} - 17)}{C_{air}(250-17)}$

Thus: $T_{H_2O_{out}} = \underline{\underline{34.58^\circ C}}$

and $Q = (34.58-17)3342 = (250 - T_{air_{out}})514.5$ so $T_{air_{out}} = \underline{\underline{135.7^\circ C}}$

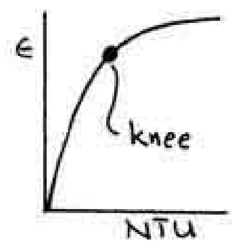
3.21 You must cool 78 kg/min of a 60%-by-mass mixture of glycerin in water from 108°C to 50°C using cooling water available at 7°C. Design a 1 shell-pass, 2 tube-pass heat exchanger if U = 637 W/m²-°C. Explain any design decisions you make and report the area, T_{H_2O}, and any other relevant features.

Since water has much the lower viscosity, we shall run it in the tube side of the exchanger.

Since water should be cheap and abundant, we shall take $C_{H_2O\text{-}glyc}$ as C_{min}. Thus:

$$\epsilon = \frac{C_h}{C_{min}} \frac{(C_{h_{in}} - T_{h_{out}})}{(T_{h_{in}} - T_{c_{in}})} = \frac{108 - 50}{108 - 7} = \underline{0.564}$$

For the best efficiency we want to operate near the knee of the effectiveness curve. To operate much to the right of the knee would mean investing in additional area but it would give little increase in ϵ. Therefore we shall select a design value of NTU = 1.5. From Fig. 3.17c we then read:



$$C_{min}/C_{max} = C_h/C_{H_2O} = \underline{0.70}$$

Then: $C_{min} = [(78/60)\text{kg/s}](2760) = 3600\ \text{J/}^\circ\text{K}$ so,

$$C_{H_2O} = 3600/0.7 = 5140 = \dot{m}_{H_2O}4177 \text{ so } \dot{m}_{H_2O} = \underline{\underline{1.23\text{ kg/s}}}$$

and the area, $A = NTU\ C_{min}/U\ 1.5(3600)/637 = \underline{\underline{8.48\ m^2}}$

Finally:

$$T_{H_2O_{out}} = T_{H_2O_{in}} + \frac{C_h}{C_c}(T_{h_{in}} - T_{h_{out}})$$

$$= 7 + \frac{3600}{5140}(108 - 50) = \underline{\underline{47.6^\circ C}}$$

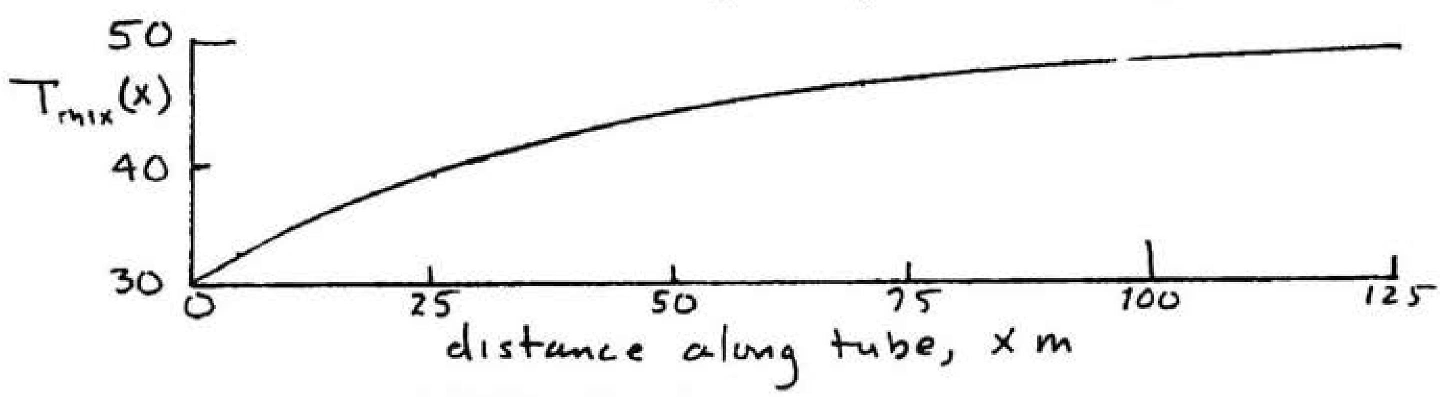
(The student designs, of course, should differ from this and from one another.)

3.22 A mixture of 40% by weight glycerin, 60% water, enters a smooth 0.113 m I.D. tube at 30°C. The tube is kept at 50°C and $\dot{m}_{mixture} = 8$ kg/s. The heat transfer coefficient inside the pipe is 1600 W/m^2-°C. Plot the liquid temperature as a function of position in the pipe.

use eqn. (3.22):
$$\epsilon = \frac{T_{mix}(x) - T_{mix,in}}{T_{pipe} - T_{mix,in}} = 1 - e^{-NTU} = 1 - \exp\left[\frac{-h}{C_{MIX}}(\pi D x)\right]$$

Where we write $U = h$ because there is only one thermal res..

Then:
$$T_{mix}(x) = 30 + (50-30)\left[1 - \exp\left(\frac{-1600}{2900(8)}\pi(0.113)x\right)\right]$$
$$= 30 + 20\left[1 - \exp(-0.02448\,x)\right]$$

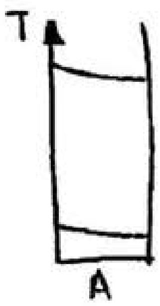


(cf. Section 7.6 of Text)

Problem 3.23 Explain in physical terms why all effectiveness curves in Fig. 3.16 and Fig. 3.17 have the same slope as NTU $\longrightarrow$ 0. Obtain this slope from eqns. (3.20) and (3.21) and give an approximate equation for Q in this limit.

Solution As NTU $\longrightarrow$ 0, the exchanger becomes too small to change the temperature of to change the temperature of either stream much at all. When the stream temperatures are nearly constant, the heat transfer is just

AT NTU → 0, the temps. of the two flows change very little.



$$Q \simeq UA\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right)$$

and so the effectiveness is just

$$\varepsilon = \frac{Q}{Q_{\max}} \simeq \frac{UA\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right)}{C_{\min}\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right)} = \frac{UA}{C_{\min}} = \text{NTU}$$

At low NTU, $\underline{\varepsilon \simeq \text{NTU}}$. This applies to any configuration, so all ε-NTU plots will have a slope of 1 for low NTU.

With respect to eqns (3.20) and (3.21), first note that

$$\exp(-x) \simeq 1 - x + \cdots \quad \text{as } x \longrightarrow 0$$

Then let NTU $\longrightarrow$ 0 in each equation.

$$\text{eqn. (3.20), parallel flow:} \quad \varepsilon \simeq \frac{1 - [1 - (1 + C_{\min}/C_{\max})\text{NTU}]}{(1 + C_{\min}/C_{\max})} = \text{NTU}$$

$$\text{eqn. (3.21), counterflow:} \quad \varepsilon \simeq \frac{1 - [1 - (1 - C_{\min}/C_{\max})\text{NTU}]}{1 - (C_{\min}/C_{\max})[1 - (1 - C_{\min}/C_{\max})\text{NTU}]} = \text{NTU}$$

Problem 3.24: We want to cool air from 150ºC to 60ºC, but we can't afford a custom-built heat exchanger. Instead, we find a used cross-flow exchanger in storage. For this one, both fluids are unmixed. It was previously used to cool 136 kg/min of NH_3 from 200ºC to 10ºC using 320 kg/min of water at 7ºC and its U was 480 W/m^{2}K.

How much air can we cool with this unit, using the same water supply, if U is about the same? (We would actually want to modify U using the methods of Chapters 6 and 7 once we had a new flow rate of air; but that's beyond our scope at the moment.)

Solution: We must first evaluate the area, based on the exchanger's previous service:

First we must evaluate the area, based on the previous service.

$$Q = (\dot{m} c_p \Delta T)_{NH_3} = \frac{136}{60} 2315 (200-100) = 524{,}733 = \underbrace{\frac{320}{60} 4196}_{C_{H_2O} = 22{,}379} (T_{H_2O_o} - 7)$$

Thus $T_{H_2O_o}$ was previously $= 30.45\ °C$, so:

$$P = \frac{30.45-7}{200-7} = 0.1215, \quad R = \frac{200-100}{30.45-7} = 4.26, \quad \text{From Fig 3.14c,} \ F = \underline{0.972}$$

Then:
$$Q = 524{,}733 = UA\ \text{LMTD}\ F = 480\, A \frac{(200-30.45)-(100-7)}{\ln(169.6/93)}\, 0.972$$

$$\underline{A = 8.828\ m^2}$$

In the new use:

$$Q = UA \cdot \text{LMTD} \cdot F, \quad \underbrace{22{,}379(T_{H_2O_o} - 7)}_{LHS} = \underbrace{480(8.828)\frac{(150 - T_{H_2O_o}) - 53}{\ln \frac{150 - T_{H_2O_o}}{53}} F}_{RHS}$$

Trial and error:

T_{H_2O}	$P = \frac{T_{H_2O_o} - 7}{150-7}$	$R = \frac{150-60}{T_{H_2O_o} - 7}$	F	LHS	RHS
25	0.126	5.00	0.965	402,822	343,135
22	0.105	6.00	0.980	335,685	353,225
22.7	0.110	5.73	0.977	351,350	351,090

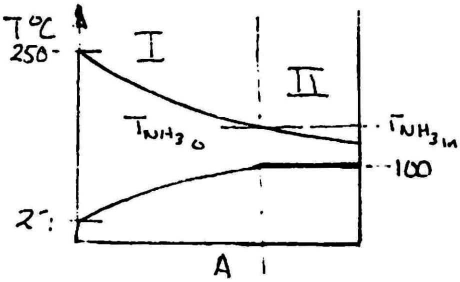
close enough

$T_{H_2O} = 22.7°F$

Finally: $Q = 22{,}379(22.7-7) = \dot{m}_{air} \underbrace{c_p}_{1011} (150-60)$

so $\dot{m}_{air} = 3.86 \frac{kg}{s} = \underline{\underline{232 \frac{kg}{min}}}$

3.25 A one tube pass, one shell pass, parallel flow, process heat exchanger cools 5kg/s of gaseous ammonia entering the shell side at 250°C and it boils 4.8kg/s of water in the tubes. The water enters subcooled at 27°C and it boils when it reaches 100°C. U = 480 W/m^2 before boiling begins and 964 W/m^2 thereafter. The area of the exchanger is 45m^2 and h_{fg} for water is 2.257 (10)^6 J/kg. Determine the quality of the water at the exit.



Consider the exchanger as two different exchangers, I & II, in series. For I, we calculate:

$C_{NH_3} = 5(2395) = 11975$ [evaluate c_p at 200°C]

$C_{H_2O} = 4.8(4180) = 20,060$

Then: $Q = 11975(250 - T_{NH_3 o}) = 20060[100-27]$, $T_{NH_3 \text{ out of I}} = \underline{127.7\,°C}$

& $\dfrac{C_{min}}{C_{max}} = 0.597$, $\varepsilon = \dfrac{250-127.7}{250-27} = 0.548$, from Fig. 3.16, LHS we

read: $NTU = 1.35 = \dfrac{UA}{C_{min}}$ so $A_I = 11975(1.35)/480 = \underline{33.68\ m^2}$

Now we are ready to deal with heat exchanger II. It has an area of $45 - 33.68 = \underline{11.32\ m^2}$ so $NTU_{II} = \dfrac{964(11.32)}{5(2270)} = \underline{0.9615}$

From equation (3.22) we get:

$$\varepsilon = 1 - e^{-NTU} = 1 - e^{-0.9615} = 0.618$$

so:

$$\frac{(127.7 - T_{NH_3 o})}{127.7 - 100} = 0.618, \quad T_{NH_3 o_{II}} = \underline{110.6\,°C}$$

Then:

$$Q = [5(2270)](127.7 - 110.6) = 194{,}190 = 2.257(10)^6\ \dot{m}_{boiled}$$

so the amount of water boiled into steam is $\underline{\dot{m}_b = 0.086\ \frac{kg}{s}}$

The exit quality is then: $x = \dfrac{0.086}{4.8} = \underline{\underline{1.8\ \%}}$

3.26 0.72kg/s of superheated steam enters a crossflow heat exchanger at 240°C and leaves at 120°C. It heats 0.6kg/s of water entering at 17°C. $U = 612\ W/m^2\text{-}°C$. By what percent will the area differ if a both-fluids-unmixed exchanger is used instead of a one-fluid-unmixed exchanger?

$Q = C_{steam}\Delta T_{steam} = 0.72(1963)(240-120) = C_{H_2O}\Delta T_{H_2O} = 0.6(4177)(T_{H_2O_{out}} - 17)$,

so $\underline{T_{H_2O} = 84.67°C}$

Now $Q = [U\,A\,LMTD]\,F$. But Q, U, and LMTD won't change so $F \sim A^{-1}$. We must next find F's.

$$P = \frac{84.67-27}{240-27} = 0.271\ ,\quad R = \frac{240-120}{84.67-27} = 2.08 \left.\right\} \begin{array}{l} PR = 0.584 \\ 1/R = 0.481 \end{array}$$

So from Fig. 3.14c, $F_{both\ unmixed} = 0.94$, $A_{both\ unmixed} \sim 1.064$

" " 3.14d, $F_{one\ mixed} = 0.923$, $A_{one\ mixed} \sim 1.083$

$$\therefore \quad \text{difference} = \frac{1.064 - 1.083}{1.083} = -0.018 = \underline{\underline{-1.8\%}}$$

1.8% less area will be needed if we make the change.

3.27 Compare values of F from Figs. 3.14c and d for the same inlet and outlet temperatures. Is the one with the highest F automatically the most desirable exchanger? Discuss.

We have Q = U·A·LMTD·F, and it's clear from Figs. 3.14c and 3.14d that for the same temperatures (the same R and P, or RP and 1/R) the resulting F is consistently lower when one flow is mixed. From the equation it's clear that to obtain a given heat flux, a higher F will require less U·A.

It follows that the exchanger with neither-flow-mixed is more desiraable on the face of it. However if P (or PR) is not large, and R (or 1/R) is, then questions as to the cost of manufacture and simplicity of arrangement will dominate. Under these circumstances, the inexpensiveness of -- say -- the exchanger in Fig. 3.6b is liable to make it economically competitive with those in Figs. 3.6 a and c, unless the performance is very greatly improved.

3.28 Compare values of ϵ for the same NTU and C_{min}/C_{max}, in parallel and counterflow heat exchangers. Is the one with the highest automatically the more desirable exchanger?

We note in Figs. 3.16, that the counterflow configuration gives a consistently higher ϵ than parallel flow does, as C_{min}/C_{max} is increased. Therefore <u>if</u> you want to transfer all of the heat that you can from one fluid to another, the counterflow arrangement <u>is</u> superior. Sometimes this is not the objective and sometimes configurational concerns are more important than a maximum ϵ. However, the counterflow arrangement does give inherently better performance.

3.29 The irreversibility rate of a process is equal to product of the rate of entropy production and the lowest absolute sink temperature accessible to the process. Calculate the irreversibility (or lost work) for the heat exchanger in Example 3.4. What kind of configuration would reduce the irreversibility, given the same end temperatures.

$$\dot{S}_{liquid} \simeq \dot{m}\int_{T_{in}}^{T_{out}} c_p \frac{dT}{T} = \dot{m}c_p \ln \frac{T_{out}}{T_{in}} \quad \text{(assuming incompressibility)}$$

$$\dot{S}_{total} = (\dot{m}c_p)_{oil} \ln \frac{T_{h\,out}}{T_{h\,in}} + (\dot{m}c_p)_{H_2O} \ln \frac{T_{c\,out}}{T_{c\,in}}\ ; \quad (\dot{m}c_p)_{H_2O} = \frac{(\dot{m}c_p)_{oil}(181-38)}{49-32}$$

$$= 5.795(2282) \ln \frac{38+273}{181+271} + \frac{5.795(2282)(181-38)}{49-32} \ln \frac{49+273}{32+273} = 1031 \frac{J}{°C\text{-}s}$$

so:

$$\dot{I} = T_o \dot{S} = (32+273)1031 = \underline{\underline{314,393\ J/s}}$$

As long as no work is delivered to the surroundings, the same temperature changes in the fluid streams will yield the same irreversibility. Another configuration cannot be used to cut our losses. Heat transfer processes simply <u>do</u> degrade energy.

PROBLEM 3.30: Plot T_{oil} and T_{H_2O} as a function of position in a very long counterflow heat exchanger where water enters at 0°C, with $C_{H_2O} = 460$ W/K, and oil enters at 90°C, with $C_{\text{oil}} = 920$ W/K, $U = 742$ W/m^{2}K, and $A = 10$ m^2. Criticize the design.

SOLUTION. The capacity-rate ratio is

$$\frac{C_{\min}}{C_{\max}} = \frac{C_{H_2O}}{C_{\text{oil}}} = \frac{460}{920} = \frac{1}{2}$$

Substituting into eqn. (3.21), we have

$$\varepsilon = \frac{1 - \exp\big[-(1 - C_{\min}/C_{\max})\text{NTU}\big]}{1 - (C_{\min}/C_{\max})\exp\big[-(1 - C_{\min}/C_{\max})\text{NTU}\big]} = \frac{1 - \exp\big[-\text{NTU}/2\big]}{1 - \frac{1}{2}\exp\big[-\text{NTU}/2\big]} \tag{1}$$

We find the water temperature from ε with eqn. (3.16), noting that water is the cold stream and oil the hot stream:

$$\varepsilon = \frac{T_{H_2O,\text{out}} - T_{H_2O,\text{in}}}{T_{\text{oil,in}} - T_{H_2O,\text{in}}}$$

To make the plot, we can imagine that the area and NTU of the exchanger are increasing as we move from the water inlet to the final outlet. Let's call the position x, so that $A(x)$ increases from zero to 10 m^2. We then write NTU in terms of $A(x)$:

$$\text{NTU}_x = \frac{UA(x)}{C_{\min}} = \frac{742\,A(x)}{460} = 1.613\,A(x) \qquad \text{for} \quad 0 \le A(x) \le 10 \text{ m}^2$$

The effectiveness accumulated up to that position is

$$\varepsilon_x = \frac{1 - \exp\big[-\text{NTU}_x/2\big]}{1 - \frac{1}{2}\exp\big[-\text{NTU}_x/2\big]} = \frac{1 - \exp\big[-0.8065A(x)\big]}{1 - \frac{1}{2}\exp\big[-0.8065A(x)\big]}$$

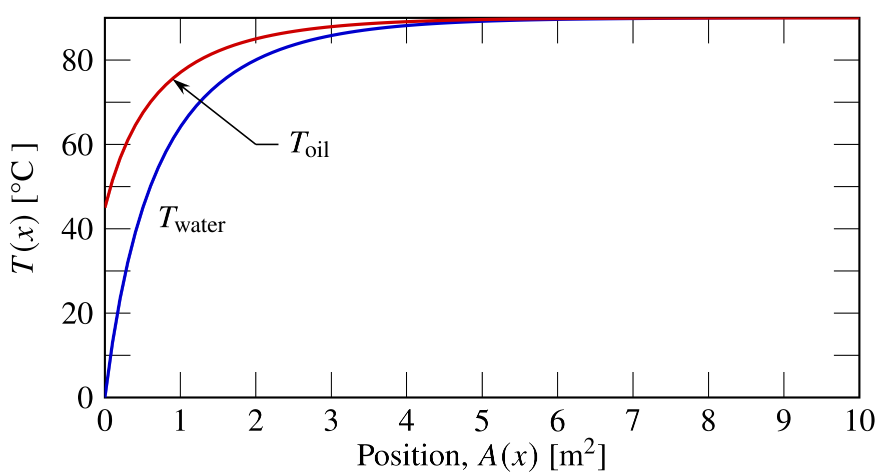
The water temperature at each location is $T_{H_2O,\,x}$. Putting $T_{H_2O,\,x}$ for $T_{H_2O,\text{out}}$, ε_x for ε, and rearranging eqn. (1)

$$T_{H_2O,\,x} = T_{H_2O,\text{in}} + \varepsilon_x\big(T_{\text{oil,in}} - T_{H_2O,\text{in}}\big) = 90\varepsilon_x \text{ °C}$$

The local temperature difference is given by eqn. (3.5b), from which the hot stream temperature is:

$$T_{\text{oil},\,x} = T_{H_2O,x} + T_{\text{oil,in}} - \left(1 - \frac{C_c}{C_h}\right)T_{H_2O,x} - \frac{C_c}{C_h}T_{H_2O,\text{out}} = 90 + \frac{T_{H_2O,\,x}}{2} - \frac{90\varepsilon_{x=10}}{2}$$

The exchanger is more than twice the size needed—right side adds nothing to the performance.



3.31 2 kg/s of liquid ammonia is cooled from 100°C to 30°C in the shell-side of a 2-shell-pass, 4-tube-pass heat exchanger, by 3 kg/s of water at 10°C. $\overline{U} = 750\ \text{W/m}^2\text{-°C}$ when the exchanger is new. Plot the exit ammonia temperature as function of the increasing tube fouling factor.

$C_{H_2O} = 4177(3) = 12{,}531\ \text{W/°C}$; $C_{NH_3} = 5282(2) = 10{,}563\ \text{W/°C} \equiv C_{min}$.

for the new exchanger, $T_{H_2O\,out} = \dfrac{10{,}563}{12{,}531}(100-30) + 10 = 69.0°\text{C}$

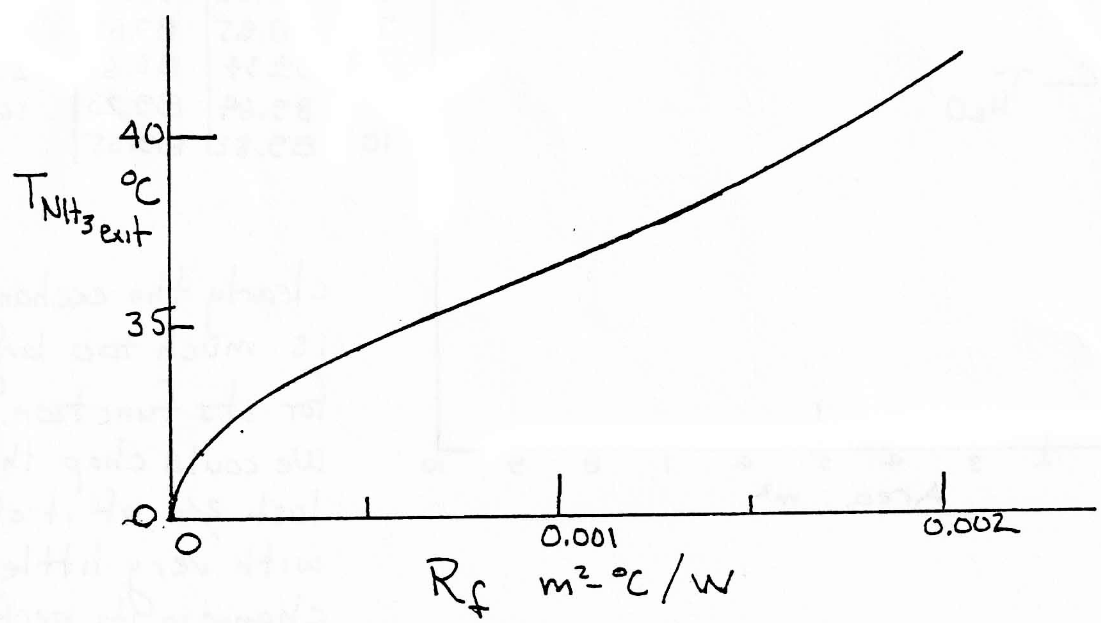
Find Area, A: $Q = 10{,}563(70) = 739{,}400\ \text{W} = \overline{U}\cdot A \cdot \text{LMTD}\cdot F$

$$\text{LMTD} = \frac{(100-69)-(30-10)}{\ln\frac{31}{20}} = 25.10, \quad F\Big(P = \underbrace{\frac{69-10}{100-10}}_{0.6556},\ R = \underbrace{\frac{100-30}{69-10}}_{1.186}\Big) = 0.61$$

So $A = Q/\overline{U}\cdot\text{LMTD}\cdot F = \underline{64.39\ \text{m}^2}$

Now we get the new exit temperatures using the effectiveness method.

R_f	$\overline{U}_{fouled}$ $\left[\frac{1}{750}+R_f\right]^{-1}$	NTU = $64.4\overline{U}_f/10563$	$\varepsilon\left(\text{NTU}, \frac{C_{min}}{C_{max}} = 0.843\right)$ from Fig. 3.17 d.)	$T_{NH_3\,out}$ = $100-(100-10)\varepsilon$
0.00005	723	4.408	0.765	31.1
0.0001	698	4.26	0.760	31.6
0.0002	652	3.98	0.755	32.1
0.0005	545	3.23	0.730	34.3
0.0010	428.6	2.613	0.710	36.1
0.0015	353	2.15	0.68	38.8
0.002	300	1.83	0.65	41.5



3.32 A 1 shell-pass, 2 tube-pass heat exchanger cools 0.403kg/s of methanol from 47°C to 7°C, on the shell side. The coolant is 2.2 kg/s of Freon 12 entering the tubes at -33°C. U = 538 W/m^2-°C. A colleague suggests that this arrangement wastes Freon. She thinks you could do almost as well if you cut the Freon flow rate all the way down to 0.8 kg/s. Calculate the new methanol outlet temperature that would result from this flow rate, and evaluate her suggestion.

Evaluate C's at $(47+7)/2 = 27°C$ for methanol and $-23°C$ for Freon:

$$C_{Freon} = 2.2(903) = 1987\ W/°C, \quad C_{methanol} = 0.403(2534) = 1021$$

Then using eqn. (3.10):

$$T_{F_{out}} = -33 + \frac{1021\ (40)}{1987} = \underline{-12.4\ °C}$$

(This gives $\bar{T}_F = (-33-12.4)/2 = -22.7°C$ so our estimate of -23 was ok.)

Now we must calculate the area, $A = Q/U \cdot LMTD \cdot F$:

$$LMTD = \frac{(47+12.4)-(7+33)}{\ln(59.4/40)} = 49.1°C$$

and at $P = \dfrac{-12.4+33}{47+33} = 0.258$, $R = \dfrac{47-7}{-12.4+33} = 1.94$] $RP = 0.501$, $1/R = 0.515$

we read $F = 0.93$ from Fig. 3.14a. Then $A = \dfrac{1021\ (40)}{538(49.1)(0.93)}$

$$\underline{A = 1.66\ m^2}$$

If we cut the coolant to 0.8 kg/s, C_F = 0.8(903) = 722.4. Then C_{min}/C_{max} = 0.708, and NTU = 538(1.66)/722.4 = 1.236. Fig. 3.17c gives ϵ = 0.56 for these parameters. Then:

$$\epsilon = 0.56 = \frac{T_{F_{out}} + 33}{47 - (-33)} = \frac{1021(47 - T_{meth_{out}})}{722.4(47 + 33)}$$

Thus: $T_{Freon_{out}} = 11.8°C$ and $T_{meth._{out}} = 15.3°C$

The suggestion looks quite good -- the methanol is only 8°C warmer. Of course U is probably smaller at the lower flow rate, so the calculation would have to be iterated.

3.33 The factors dictating the heat transfer coefficients in a certain 2-shell-pass, 4-tube-pass heat exchanger are such that U increases as $(\dot{m}_{shell})^{0.6}$. The exchanger cools 2 kg/s of air from 200°C to 40°C using 4.4 kg/s of water at 7°C and U = 312 W/m^2-°C under these circumstances. If we double the air flow, what will its temperature be, leaving the exchanger?

$C_{air} = 2(1012) = 2024$ W/°C , $C_{H_2O} = 4.4(4177) = 18{,}379$ W/°C

(cp is pretty temperature both fluids. Correction will probably not be needed.)

Find A. To do this, first find $T_{H_2O_{out}}$. $\Delta T_{H_2O} = \dfrac{C_{air}\,\Delta T_{air}}{C_{H_2O}} = \dfrac{2024(160)}{18{,}329}$

so: $\Delta T_{H_2O} = 17.67$; then $T_{H_2O_{out}} = 7 + 17.67 = \underline{24.67°C}$

Then $A = \dfrac{Q}{U \cdot LMTD \cdot F}$: $LMTD = \dfrac{(200-24.67)-(40-7)}{\ln(175.33/33)} = \underline{85.22°C}$

& at $P = \dfrac{24.67-7}{200-7} = 0.0916$, $R = \dfrac{200-40}{17.67} = 9.05$ } $RP = 0.829$, $1/R = 0.1105$

so from Fig. 3.14b we read $\underline{F = 0.98}$

$A = 2024(160)/312(85.22)(0.98) = \underline{12.43\text{ m}^2}$

Now the new U is $312(2)^{0.6} = 473$ W/m^2-°C. So the new NTU is $U_{new}A/C_{min_{new}} = 473(13.43)/2024(2) = 1.45$ and $(C_{min}/C_{max})_{new} = 2(2024)/18{,}379 = 0.2203$. Thus Fig. 3.17c gives:

$$\varepsilon = 0.72 = \frac{200 - T_{air_{out}}}{200 - 7}$$

so the revised temperature of air out is $T_{air_{out}} = \underline{\underline{61.0°C}}$.

3.34 A flow rate of 1.4 kg/s of water enters the tubes of a 2-shell-pass, 4-tube-pass heat exchanger at 7°C. A flow rate of 0.6 kg/s of liquid ammonia at 100°C is to be cooled to 30°C on the shell side. U=573 W/m^2-°C. a.) How large must the heat exchanger be? b.) How large must it be if, after some months, a fouling factor of 0.0015 $\frac{W}{m^2\text{-}°C}$ will build up in the tubes, and we still want to deliver ammonia at 30°C? c.) If we make it large enough to accommodate fouling, what temperature will it cool the ammonia to when it is new? d.) What temperature does water leave the new enlarged exchanger

$C_{H_2O} \simeq 1.4(4190) = \underline{5866 \text{ W/°C}}$; $C_{NH_3} = 0.6(5189) = \underline{3113 \text{ W/°C}}$

Then $C_{min}/C_{max} = 0.531$ & we want ε to be $\frac{100-30}{100-7} = 0.753$. Thus we read from Fig. 3.17d, $NTU = 2.1 = UA/C_{min} = 573A/3113$.

$\underline{\underline{A = 11.41 \text{ m}^2}}$ ← a)

$$U_{fouled} = \frac{1}{R_f + \frac{1}{U_{new}}} = \underline{308 \text{ W/m}^2\text{-°C}}$$, but NTU is still 2.1. Thus:

$2.1 = \frac{308 A}{3113.4}$, $\underline{\underline{A = 21.22 \text{ m}^2}}$ ← b)

With this area, $NTU_{new} = \frac{573(21.22)}{3113} = 3.905$ so $\varepsilon = 0.87$

Then $0.87 = \frac{100 - T_{NH_{3o}}}{100-7} = \frac{5866}{3113}\frac{T_{H_2O_o} - 7}{100-7}$ ∴ $T_{NH_{3\,out}} = 19.1°C$ ← c)

$\underline{\underline{T_{H_2O_{out}} = 49.9°C}}$ ← d)

If we design the exchanger to function after fouling has occurred, it will undercool the ammonia by 11°C. If this should be unacceptable, then the water flow rate will have to be controlled.

[We could also calculate the water temperature leaving the old fouled exchanger: $NTU = \frac{11.41(308)}{3100} = 1.13$ so $\varepsilon = 0.63$

$$0.63 = \frac{5866}{3133}\,\frac{T_{H_2O_o} - 7}{100-7}$$

so $T_{H_2O_{out}} = \underline{38.1°C}$]

3.35 Equation (3.21) is troublesome when $C_{min}/C_{max} = 1$. Develop a working equation for ε in this case. Compare it with Fig. 3.16.

Call $C_{min}/C_{max} \equiv x$. Then: $\varepsilon = \dfrac{1-\exp(-[1-x]\,NTU)}{1-x\exp(-[1-x]NTU)}$ which goes to $\frac{0}{0}$ as $x \Rightarrow 1$. Thus ε becomes indeterminate and we resort to L'Hospital's rule:

$$\lim_{x\Rightarrow 1}\varepsilon = \left.\frac{d(\text{Num.})/dx}{d(\text{Denom.})/dx}\right|_{x=1} = \left.\frac{-NTU\exp(-[1-x]NTU)}{-\exp(-[1-x]NTU) - x\,NTU\exp(-[1-x]NTU)}\right|_{x=1}$$

so:

$$\varepsilon = \frac{NTU}{1+NTU} \quad \text{when } C_{min}/C_{max} = 1$$

Calculated points $\varepsilon(NTU=1) = 0.5$, $\varepsilon(NTU=2) = 2/3$, $\varepsilon(NTU=3) = 3/4$, ·· etc. match the $C_{min}/C_{max} = 1$ line in Fig. 3.16, perfectly.

Problem 3.36: Both *C*'s in a parallel-flow heat exchanger are equal to 156 W/K, $U = 327$ W/m^2 K, and $A = 2$ m^2. The hot fluid enters at 140°C. If we cut both *C*'s in half, what will the exit temperature of the hot fluid be?

Solution: NTU = 327(2)/156 = 4.19 and C_{min}/C_{max} is 1.00. Cutting the *C*'s in half will make the NTU still larger. We see, in Fig. 3.16, that ε is constant in this NTU range. Thus, the exiting hot water temperature is unchanged:

$T_{hot\text{-}out} = 90$ °C

Problem 3.37: A 1.68 ft^2 crossflow heat exchanger with one fluid mixed condenses steam at atmospheric pressure (h = 2000 Btu/ft^2-hr-°F) and boils methanol (T_{sat} = 170 °F and h = 1500 Btu/ft^2-hr-°F) on the other side. Evaluate *U* (neglecting the metal's resistance), *F*, LMTD, & *Q*. Can we evaluate NTU and ε?

$U = [1/2000 + 1/1500]^{-1} =$ 857 Btu/ft^2-hr-°F

LMTD (per Example 3.2) = $T_{steam} - T_{methanol}$ = 212 -170 = 42 °F

From Fig. 3.14d, *F* for *P* = 0 and any *R* is equal to 1.0

So, using eqn. (3.14), $Q = UA\Delta TF$ = (857)(1.68)(42)(1) = 60,470 Btu/hr

NTU and ε are not meaningful, since neither C_{min} nor C_{max} is known or relevant. Flow rates have no bearing on *Q* in this case. This configuration is a simple case of conduction through a wall with two significant resistances.

Problem 3.38 We can calculate the effectiveness of a crossflow heat exchanger with neither fluid mixed using the approximate formula:

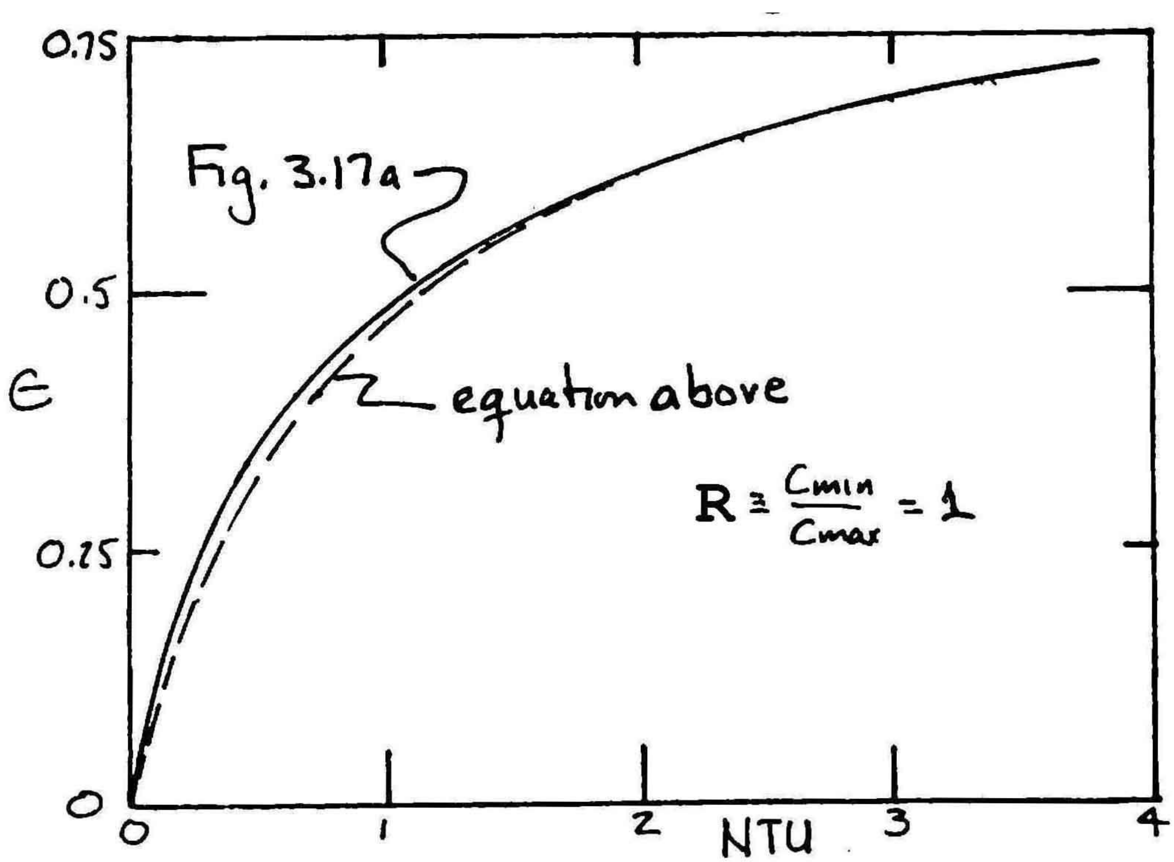
$$\varepsilon \approx 1 - \exp\{[\exp(-NTU^{0.78}R) - 1][NTU^{0.22}/R]\}$$

where $R \equiv C_{min}/C_{max}$. How closely does this correspond to exact results for known limiting cases? Present results graphically.

Solution As R goes to 0, ε therefore approaches $\underline{\underline{1 - \exp(-NTU)}}$

[This is exactly the single stream result, eqn. (3.22)]

We evaluate the equation numerically for R = 1, and compare it with Fig. 3.17a in the following graph. It shows this approximation to be very good at this value of R. And it will approach being exactly the same, as R is lowered.



3.39 Calculate the area required in a 2 tube pass, 1 shell pass, condenser that is to condense 10^6 kg/hr of steam at 40°C using water at 17°C. $U = 4700\ W/m^2\text{-}°C$, the maximum allowable temperature rise of the water is 10°C, and $h_{fg} = 2406$ kJ/kg.

$$\epsilon = \frac{27-17}{40-17} = 0.435 = 1 - e^{-NTU} \qquad \text{(eqn. 3.22)}$$

Therefore: $NTU = \dfrac{UA}{C_{min}} = 0.5705$

But $C_{min} = \dot{m}c_p = \dfrac{Q}{\Delta T} = \dfrac{(10^6/3600)(2{,}406{,}000)}{10} = 6.683\,(10)^7\ \dfrac{W}{°C}$

so: $\text{Area} = \dfrac{C_{min}\,NTU}{U} = \dfrac{6.683(10)^7(0.5705)}{4700} = \underline{\underline{8112\ m^2}}$

3.40 An engineer wants to divert 1 gpm of water at 180°F from his car radiator, through a small cross-flow heat exchanger with neither flow mixed, to heat 40°F water to 140°F for shaving when he goes camping. If he wants to produce 1 pint per minute of hot water, what will be the area of the exchanger and the temperature of the returning radiator coolant if U is 720 $W/m^2\text{-}°C$?

We evaluate water properties at an average temp. of 90°F = $\underline{32°C}$.

$$C_h = (\dot{m}c_p)_{H_2O} = 8C_c = \left[993.5\,\frac{kg}{m^3}\left(0.06243\,\frac{lb_m/ft^3}{kg/m^3}\right)\right]\left[\frac{gal}{60\,sec}\ \frac{231\,in^3}{gal}\ \frac{ft^3}{1728\,in^3}\right]$$
$$\times\left[4197\,\frac{J}{kg°C}\ 0.23884\,\frac{Btu/lb_m°F}{J/kg°C}\right] = \underline{138.5\ \frac{Btu}{°R}}$$

$T_{h_{out}} = 180 - \frac{1}{8}(140-40) = \underline{\underline{167.5\,°F}}$

Then the effectiveness is: $\epsilon = \dfrac{140-40}{180-40} = \underline{0.714}$ so from Fig. 3.17a we read

$NTU = \underline{0.49}$

But

$$NTU = \frac{UA}{C_{min}} = \frac{[720(0.1761)]A}{[138.5/8]} = 0.49$$

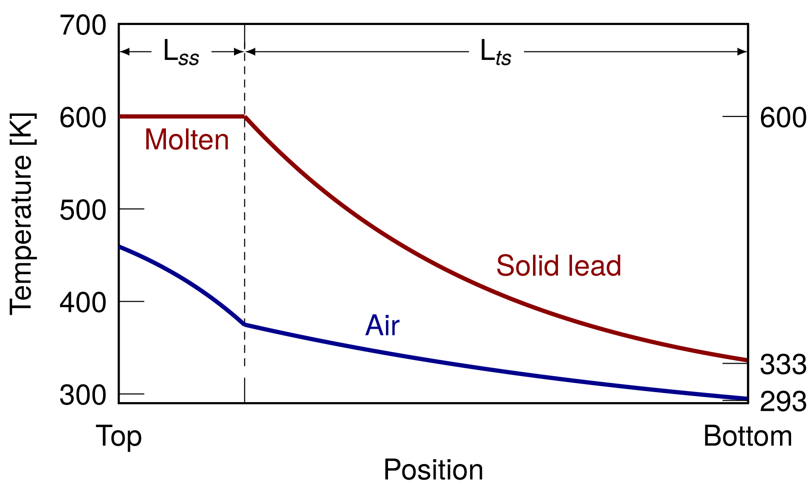
Therefore we obtain: $A = 0.067\ ft^2 = \underline{\underline{0.00622\ m^2}}$

(This is really quite small. It would probably pay to increase the size & up the production of hot water.)

Problem 3.41 To make lead shot, molten droplets of lead are showered into the top of a tall tower. The droplets fall through air and solidify before they reach the bottom of the tower, where they are collected. Cool air is introduced at the bottom of the tower and warm air flows out the top. For a particular tower, 5,000 kg/hr of 2.8 mm diam. droplets are released at their melting temperature of 600 K. The latent heat of solidification is 23.1 kJ/kg. The dropping pan size produces 6,700 droplets/m^3 in the tower. Air enters the bottom at 20 °C with a mass flow rate of 2,400 kg/hr. The tower has an internal diameter of 0.6 m with adiabatic walls.

a) Sketch, qualitatively, the temperature distributions of the shot and the air along the height of the tower.
b) If it is desired to remove the shot at a temperature of 60 °C, what will be the temperature of the air leaving the top of the tower?
c) Determine the air temperature at the point where the lead has just finished solidifying.
d) Determine the height that the tower must have in order to function as desired. The heat transfer coefficient between the air and the droplets is $\overline{h} = 170$ W/m^{2}K.

Solution (a) This is a counterflow heat exchanger. The lead shot is the hot stream and the air is the cold stream. However, while the lead is solidifying, it remains at its freezing temperature. The small metal beads will be isothermal (a calculation shows that the Biot number is $\ll 1$).



(b) The energy given up by the lead goes to the air.

$$(\dot{m}c_p)_{\text{air}}(T_{\text{air},o} - T_{\text{air},i}) = \dot{m}_{\text{lead}}\left[h_{sf} + c_{p,\text{lead}}(T_{\text{lead},i} - T_{\text{lead},o})\right]$$

$$(2400)(1008)(T_{\text{air},o} - 20) = (5000)\,[23100 + (148)(600 - 333)]$$

Solving, $T_{\text{air},o} = \underline{149.4\text{ °C}}$.

(c) We may consider the air between the freezing point and the outlet:

$$(\dot{m}c_p)_{\text{air}}(T_{\text{air},o} - T_{\text{air,solid}}) = \dot{m}_{\text{lead}}h_{sf}$$

$$(2400)(1008)(149.4 - T_{\text{air, solid}}) = (5000)(23100)$$

Solving, $T_{\text{air,solid}} = \underline{101.7\text{ °C}}$.

(d) The height must be determined in two pieces. One is a single stream exchanger with lead at its freezing temperature. The other is a counterflow exchanger for the section in which the solid lead cools. For each section, we need to find the NTU that gives that section the effectiveness needed to attain the indicated temperatures. The NTU determines the contact area and therefore the height of each section.

For both sections the overall heat transfer coefficient is simply the air-side heat transfer coefficient, since the lead beads have negligible thermal resistance: $U = 170$ W/m^{2}K. The surface area per meter height is

$$A = \big(6700\big)\pi(0.0028)^2 \frac{\pi}{4}(0.6)^2 = 0.0467 \text{ m}^2\text{/m}$$

For the single-stream side, with $C_{\text{min}} = C_{\text{air}}$, the effectiveness is

$$\varepsilon = \frac{T_{\text{air},o} - T_{\text{air,solid}}}{T_{\text{lead},i} - T_{\text{air},i}} = \frac{149.4 - 101.7}{600 - 293} = 0.1554$$

We can solve eqn. (3.22) for NTU = 0.1688. If the single-stream side has length L_{ss}

$$\text{NTU}_{\text{ss}} = \frac{UA}{C_{\text{min}}} = \frac{(170)(0.0467)L_{\text{ss}}}{(2400/3600)(1008)} = 0.1688$$

so that $L_{\text{ss}} = 14.3$ m.

For the two-stream section, $C_{\text{min}} = C_{\text{lead}}$, and

$$\varepsilon = \frac{T_{\text{lead},i} - T_{\text{lead},o}}{T_{\text{lead},i} - T_{\text{air},i}} = \frac{600 - 333}{600 - 293} = 0.870$$

With $C_{\text{min}}/C_{\text{max}} = (5000)(148)/(2400)(1008) = 0.306$, we may read from Fig. 3.16: NTU = 2.5. (Substitution into eqn. (3.21) confirms this result.) Solving

$$\text{NTU}_{\text{ts}} = \frac{UA}{C_{\text{min}}} = \frac{(170)(0.0467)L_{\text{ts}}}{(5000/3600)(148)} = 2.5$$

so that $L_{\text{ts}} = 64.7$ m.

The total height of the tower, L, is

$$L = L_{\text{ss}} + L_{\text{ts}} = 14.3 + 64.7 = \underline{79 \text{ m}}$$

Problem 3.42 The entropy change per unit mass of a fluid taken from temperature T_i to temperature T_o at constant pressure is $s_o - s_i = c_p \ln(T_o/T_i)$ in J/K·kg. (a) Apply the Second Law of Thermodynamics to a control volume surrounding a counterflow heat exchanger to determine the rate of entropy generation, $\dot{S}_{\text{gen}}$, in W/K. (b) Write $\dot{S}_{\text{gen}}/C_{\min}$ as a function of ε, the heat capacity rate ratio, and $T_{h,i}/T_{c,i}$. (c) Show (e.g., by plotting) that $\dot{S}_{\text{gen}}/C_{\min}$ is minimized when $C_{\min} = C_{\max}$ (balanced counterflow) for fixed values of ε and $T_{h,i}/T_{c,i}$.

Solution (a) The entropy generation is just the difference between the entropy carried out by the flows and the entropy carried in by the flows, so

$$\dot{S}_{\text{gen}} = \dot{m}_h \Delta s_h + \dot{m}_c \Delta s_c = (\dot{m} c_p)_h \ln\left(\frac{T_{h,o}}{T_{h,i}}\right) + (\dot{m} c_p)_c \ln\left(\frac{T_{c,o}}{T_{c,i}}\right) = \underline{C_h \ln\left(\frac{T_{h,o}}{T_{h,i}}\right) + C_c \ln\left(\frac{T_{c,o}}{T_{c,i}}\right)} \qquad (1)$$

Note that the entropy of the hot stream decreases.

(b) We'll need to distinguish between the cases when $C_h > C_c = C_{\min}$ and $C_c > C_h = C_{\min}$ when using eqn. (3.16). For $C_h > C_c$:

$$T_{h,o} = T_{h,i} - \varepsilon \frac{C_c}{C_h}(T_{h,i} - T_{c,i})$$

and

$$T_{c,o} = T_{c,i} + \varepsilon (T_{h,i} - T_{c,i})$$

Substituting into eqn. (1) gives

$$\frac{\dot{S}_{\text{gen}}}{C_{\min}} = \frac{C_h}{C_c} \ln\left[1 - \varepsilon \frac{C_c}{C_h}\left(1 - \frac{T_{c,i}}{T_{h,i}}\right)\right] + \ln\left[1 + \varepsilon\left(\frac{T_{h,i}}{T_{c,i}} - 1\right)\right] \qquad (2)$$

For the case $C_h < C_c$:

$$T_{h,o} = T_{h,i} - \varepsilon (T_{h,i} - T_{c,i})$$

and

$$T_{c,o} = T_{c,i} + \varepsilon \frac{C_h}{C_c}(T_{h,i} - T_{c,i})$$

Substituting into eqn. (1) as before gives

$$\frac{\dot{S}_{\text{gen}}}{C_{\min}} = \ln\left[1 - \varepsilon\left(1 - \frac{T_{c,i}}{T_{h,i}}\right)\right] + \frac{C_c}{C_h} \ln\left[1 + \varepsilon \frac{C_h}{C_c}\left(\frac{T_{h,i}}{T_{c,i}} - 1\right)\right] \qquad (3)$$

Both eqn. (2) and (3) have the form:

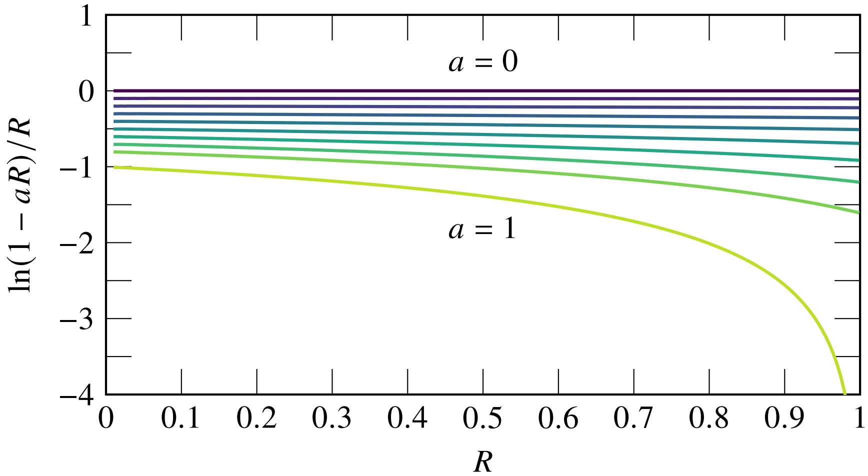
$$\frac{\dot{S}_{\text{gen}}}{C_{\min}} = \underline{\text{fn}\left(\varepsilon, \frac{C_h}{C_c}, \frac{T_{h,i}}{T_{c,i}}\right)}$$

(c) For $R = C_{\min}/C_{\max}$. Then $0 < R \leqslant 1$. For $C_h > C_c$:

$$\frac{\dot{S}_{\text{gen}}}{C_{\min}} = \frac{1}{R} \ln\left[1 - R\, \underbrace{\varepsilon\left(1 - \frac{T_{c,i}}{T_{h,i}}\right)}_{\equiv a,\ \text{constant}>0}\right] + \underbrace{\ln\left[1 + \varepsilon\left(\frac{T_{h,i}}{T_{c,i}} - 1\right)\right]}_{\equiv b,\ \text{constant}>0}$$

$$= \frac{1}{R} \ln(1 - aR) + b$$

The easiest way to think about this function is to plot the first term for a few values of a, noting that $0 \leqslant a \leqslant 1$. From the following plot, it's clear that the lowest values of $\dot{S}_{\text{gen}}/C_{\min}$ will be at $R = 1$.

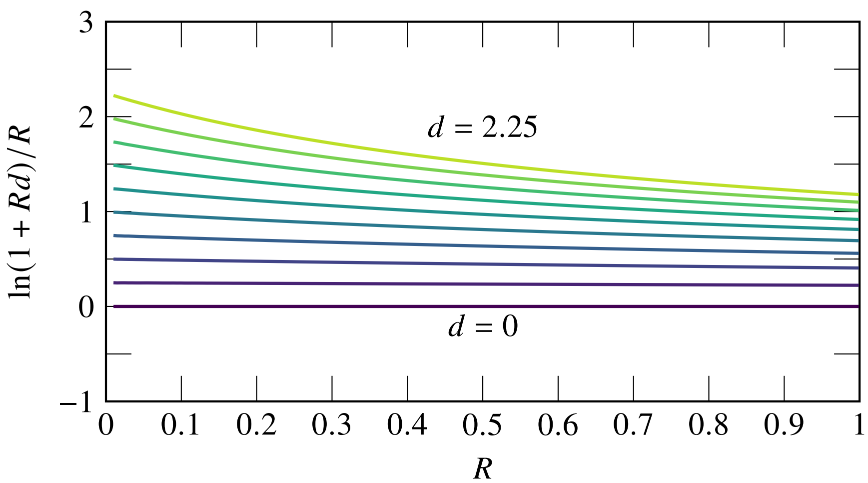


For $C_h < C_c$:

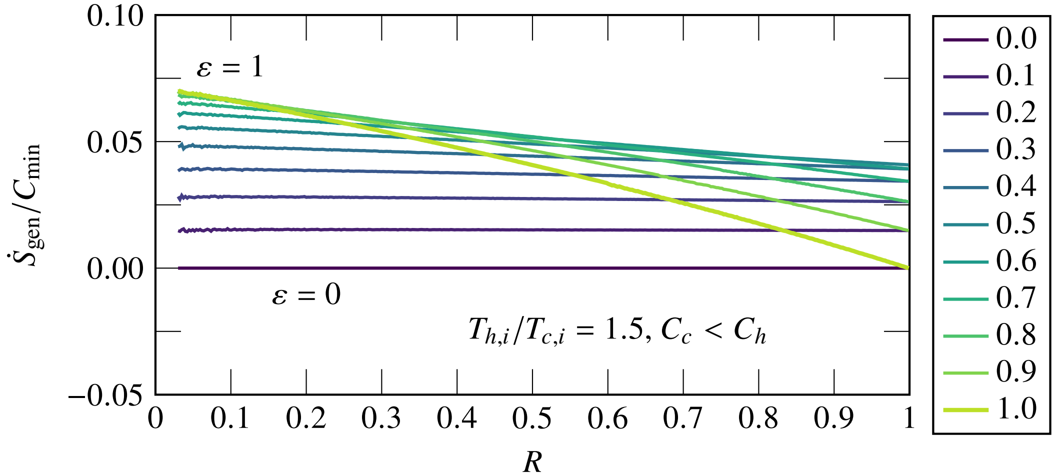
$$\frac{\dot{S}_{\text{gen}}}{C_{\min}} = \underbrace{\ln\left[1 - \varepsilon\left(1 - \frac{T_{c,i}}{T_{h,i}}\right)\right]}_{\equiv c,\text{ constant}<0} + \frac{1}{R}\ln\left[1 + R\,\varepsilon\underbrace{\left(\frac{T_{h,i}}{T_{c,i}} - 1\right)}_{\equiv d,\text{ constant}>0}\right]$$

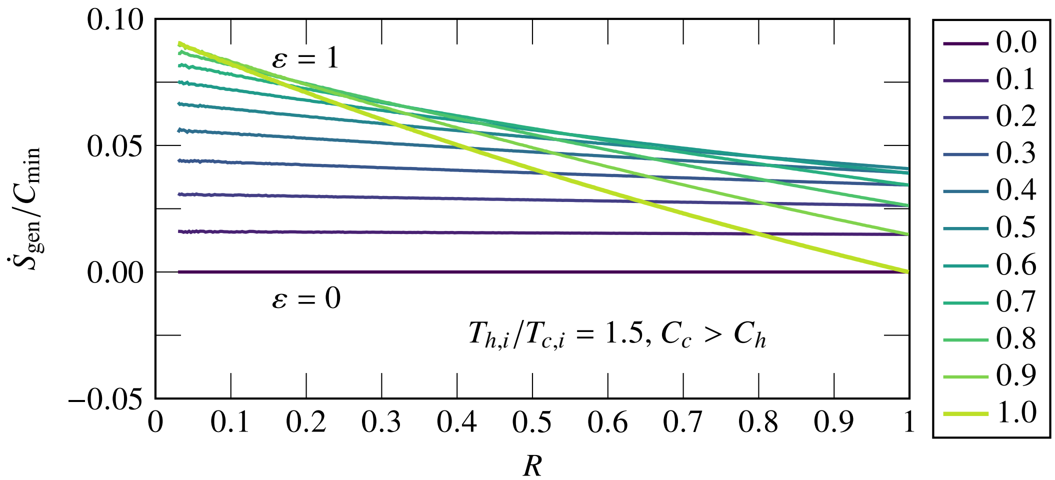
$$= c + \frac{1}{R}\ln(1 + Rd)$$

Plotting the relevant part of this expression, we see that it also has a minimum at $R = 1$.



Another way to approach this is to plot $\dot{S}_{\text{gen}}/C_{\min}$ for fixed $T_{h,i}/T_{c,i}$ and several values of ε.





We can see clearly that balancing the exchanger, so that $C_{\max} = C_{\min}$, minimizes the entropy generation rate for a given effectiveness and given inlet temperatures. (Note that fixing the effectiveness does *not* fix the size of the heat exchanger: for any operating point, the exchanger would need to be sized so as to provide the desired effectiveness.)

REFERENCE: G.P. Narayan, J.H. Lienhard V, and S.M. Zubair, "Entropy Generation Minimization of Combined Heat and Mass Transfer Devices," *Int. J. Thermal Sciences*, **49**(10):2057-2066, Oct. 2010.

Problem 3.43 Entropy generation in a power cycle lowers efficiency relative to the Carnot efficiency. Heat exchangers contribute to this loss. As seen in Problem 3.42, balanced counterflow heat exchangers can help to limit entropy generation. Let's look at the entropy generation of a balanced exchanger.

a) Let $\Delta T = T_h - T_c \ll T_{\text{c, in}}$ (in kelvin). Show that the entropy generation rate in a small area $dA = P dx$ (with P the perimeter) of the exchanger is

$$d\dot{S}''_{\text{gen}} = dQ\left(\frac{1}{T_c} - \frac{1}{T_h}\right) \simeq \frac{UP\Delta T^2}{T_c^2}\,dx$$

b) Show that the total entropy generation rate is

$$\dot{S}_{\text{gen}} \simeq Q\left(\frac{\Delta T}{T_{\text{h,in}}\,T_{\text{c,in}}}\right)$$

c) If a fixed heat load, Q, needs to be transferred, how can entropy generation be reduced? Discuss how cost and other considerations affect your answer.

Solution (a) Equation (1.7) gives the rate entropy generation when heat flow from one temperature to another. For a heat transfer per unit area of dQ (W/m^2) going from T_h to T_c, eqn. (1.7) becomes

$$d\dot{S}''_{\text{gen}} = dQ\left(\frac{1}{T_c} - \frac{1}{T_h}\right)$$

If $\Delta T = T_h - T_c$ and $\Delta T \ll T_{\text{c, in}}$, then

$$\left(\frac{1}{T_c} - \frac{1}{T_h}\right) = \left(\frac{1}{T_c} - \frac{1}{T_c + \Delta T}\right) = \left(\frac{1}{T_c} - \frac{1}{T_c}(1 - \Delta T/T_c + \cdots)\right) \simeq \frac{\Delta T}{T_c^2}$$

With $dQ = U\Delta T\, dA$ from eqn. (3.2) and $dA = P dx$, we get

$$d\dot{S}''_{\text{gen}} \simeq dQ\,\frac{\Delta T}{T_c^2} = \frac{UP\Delta T^2}{T_c^2}\,dx \tag{1}$$

(b) For a balanced counterflow exchanger, the temperature difference between hot and cold streams is constant through the whole length of the exchanger (because $C_c = C_h$, the streams have the same temperature change in response to heat transfer between them). Therefore, the temperature varies as a straight line from inlet ($x = 0$) to outlet ($x = L$) with a slope $a = (T_{c,\text{out}} - T_{c,\text{in}})/L$:

$$T_c(x) = T_{c,\text{in}} + ax/L$$

Putting this into eqn. 1 and integrating from 0 to L

$$\dot{S}_{\text{gen}} \simeq \int_0^L \frac{UP\Delta T^2}{(T_{c,\text{in}} + ax/L)^2}\,dx = \frac{UP\Delta T^2}{a}\left(\frac{1}{T_{c,\text{in}}} - \frac{1}{T_{c,\text{out}}}\right) = \left(\frac{Q\Delta T}{T_{c,\text{in}}T_{c,\text{out}}}\right) \simeq \left(\frac{Q\Delta T}{T_{c,\text{in}}T_{h,\text{in}}}\right) \tag{2}$$

where the last step follows from $T_{h,\text{in}} = T_{c,\text{out}} + \Delta T \simeq T_{c,\text{out}}$. (Recall from Example 3.2 that a balanced exchanger has LMTD = ΔT so that $Q = UA\,\Delta T$.)

(c) From eqn. (2), entropy generation can only be reduced if ΔT is reduced. Holding $Q = UA\,\Delta T$ fixed, a reduction in ΔT can be achieved by increasing the area or by increasing U. Increasing the area means a larger heat exchanger and greater capital cost. We'll see in Chapter 7 that increasing U at fixed flow rate generally means roughening the surface or reducing the size fluid passages; but those changes can increase pressure drop, susceptibility to fouling, and/or cost.

PROBLEM 3.44 Water at 100 °C flows into a bundle of 30 copper tubes. The tubes are 28.6 mm O.D. and 3 m long with a wall thickness of 0.9 mm. Air at 20 °C flows into the bundle, perpendicular to the tubes. The mass flow rate of water is 17 kg/s and that of air is 25 kg/s. (a) Determine the outlet temperature of the water if $\overline{h}_{\text{water}} = 7200$ W/m^{2}K and $\overline{h}_{\text{air}} = 110$ W/m^{2}K. (b) To improve the heat removal, aluminum fins are placed on the outside of the tubes (see Fig. 3.6b). The surface area of the fins and tubes together is now 81 m^2. Explain in words why the fins improve heat removal. If the conduction resistance of the fins is small and $\overline{h}_{\text{air}}$ is unchanged, what is the new outlet temperature of the water? *Hint:* See Problem 3.38.

SOLUTION (a) This cross-flow heat exchanger has the water stream unmixed. First find UA:

$$
\begin{aligned}
UA &= (R_{\text{water}} + R_{\text{tube}} + R_{\text{air}})^{-1} \\
&= \left(\frac{1}{\overline{h}_{\text{water}}(30\pi D_i L)} + \frac{t}{30k\pi DL} + \frac{1}{\overline{h}_{\text{air}}(30\pi D_o L)} \right)^{-1} \\
&= 30\pi(3) \left(\frac{1}{7200(0.0268)} + \frac{0.0009}{(396)(0.0277)} + \frac{1}{(110)(0.0286)} \right)^{-1} \\
&= 282.7 \left(5.182 \times 10^{-3} + 8.205 \times 10^{-5} + 3.179 \times 10^{-1} \right)^{-1} = 874.9 \text{ W/K}
\end{aligned}
$$

Observe that the air-side resistance is the largest by two orders of magnitude, and that the tube wall resistance is entirely negligible.

We have $C_h = (\dot{m}c_p)_{H_2O} = (17)(4210) = 7.16 \times 10^4$ W/K and $C_c = (\dot{m}c_p)_{\text{air}} = (25)(1007) = 2.52 \times 10^4$ W/K. The NTU is

$$
\text{NTU} = \frac{UA}{C_c} = \frac{874.9}{2.52 \times 10^4} = 0.0347
$$

This is very, very low! Figure 3.17b shows that ε will be tiny. Neither stream will experience much change in temperature, and <u>the water leaves at about 100 °C</u>.

(b) Fins are added in order to increase the area on the air-side, thereby lowering the air-side thermal resistance. If the conduction resistance of the fins is negligible, they are isothermal at the tube surface temperature. (In Section 4.5, we show how to calculate a fin's conduction resistance.)

$$
\begin{aligned}
UA &= \left(\frac{1}{(7200)30\pi(0.0268)(3)} + \frac{0.0009}{(396)30\pi(0.0277)(3)} + \frac{1}{(110)(81)} \right)^{-1} \\
&= \left(1.833 \times 10^{-5} + 2.902 \times 10^{-7} + 1.122 \times 10^{-4} + \right)^{-1} = 7,642 \text{ W/K}
\end{aligned}
$$

The NTU is

$$
\text{NTU} = \frac{UA}{C_c} = \frac{7642}{2.52 \times 10^4} = 0.303
$$

In this case, neither stream is mixed, and $r = C_{\min}/C_{\max} = 0.352$. With Fig. 3.17a, $\varepsilon \approx 0.25$. More precisely, we may use the equation given in Problem 3.38:

$$
\varepsilon = 1 - \exp\left\{ \left[\exp\left(-(0.303)^{0.78}(0.352)\right) - 1 \right] (0.303)^{0.22} / (0.352) \right\} = 0.246
$$

Using eqn. (3.16), we find

$$
T_{\text{water, out}} = T_{h_{\text{in}}} - \varepsilon \frac{C_c}{C_h}(T_{h_{\text{in}}} - T_{c_{\text{in}}}) = 100 - (0.246)(0.352)(100 - 20) = \underline{93.1 \text{ °C}}
$$